The Tax-Foundation Theory of Money

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Abstract

Throughout modern history governments have tried to promote the general acceptance of their unbacked paper currencies. One of the most common devices has been legal tender laws that have assured the acceptance of these currencies as tax payments. I provide the first dynamic mathematical model of this mechanism, thus showing explicitly the medium of exchange role of money, accounting for expectations about the government's survival, and enabling more realistic taxation systems. I show that whether competing with other paper moneys or checks, a stable government can promote its currency by refusing to accept the other objects in tax payments. While this mechanism has similarities to convertibility, it differs from it on a critical aspect: with this mechanism the government can often keep its favorite money in circulation even while increasing its quantity and thus causing it to decrease in value. This opens the door for an effective inflationary policy. The theory is shown to be consistent with numerous failures of government-issued moneys but also with their more numerous successes.

"In practice credible sovereign power—specifically, the ability to enforce the legal tender status of fiat money—is necessary to create the expectations that support a viable fiat money. (Recall that the defeat of the Confederacy rendered Confederate fiat money worthless). ... The reconciliation of theory with 'facts' about fiat money remains a central problem in monetary economics."

Herschel Grossman (1991)

1 Introduction

The circulation of inconvertible, intrinsically useless money is a fundamental puzzle in monetary theory. Standard models, such as the random matching model and the overlapping generations model, attribute it to self-fulfilling expectations. In these models there is always an equilibrium in which such money does not circulate because nobody believes that it will. This equilibrium can only be eliminated by introducing an external entity, i.e., a government. The government can simply force agents to accept money in trade.[1] Alternatively, if the government itself accepts the money in trade, this can induce agents to do the same.[2] However, the application of either mechanism to modern capitalistic democracies is questionable. In most of these countries the government's favorite money is not forced on spot transactions. Although some believe that legal tender laws force money on all transactions, this is clearly not true, as central banks openly admit (see below). As for government sales in the marketplace (not including "free" public services), in many countries they amount to nothing more than sale of postal stamps.

This chapter explores an alternative mechanism of government intervention. The government chooses which objects to accept in tax payments and thus affects the value of these objects and their potential to circulate as media of exchange. Unlike the above-mentioned mechanisms this one is very realistic, appearing in every modern legal tender law. It also has a distinguished past. It was used as the legal support of paper money in Medieval China (von Glahn 1996). It was introduced in the West by Massachusetts in 1690 (Goldberg 2009) and copied from there to the rest of the world.

In contrast to the prevalence of this mechanism in reality, mainstream economic theorists have largely ignored the potential and significance of this mechanism. Succinctly named "the tax-foundation theory" by Ellis (1934, p. 11), it was briefly discussed in mainstream economics by Smith (1776) and Lerner (1947).[3] With the current emphasis on increasingly sophisticated mathematical models, this theory has been somewhat neglected. Charles Goodhart criticized economists for being attached to "nicely constructed models, whatever the facts may be" even though the tax-foundation theory "does far better

[1] E.g., Lotz and Rocheteau (2002), Selgin (2003).

[2] E.g., Aiyagari and Wallace (1997), Li and Wright (1998).

[3] Wray (1998) and Forstater (2006) survey the history of economic thought on this theory.

in explaining and predicting historical reality" (Goodhart 1998, p. 408-9). There have been only two attempts so far to formalize the theory in a model, by Starr (1974, 2003). Starr's models are static. The former is a Walrasian model where cash is assumed to be necessary for consumption, while the latter is a trading post model. In both models all agents are taxed with probability one. It may be argued that these models—being static—only show that money has value but not that it is a medium of exchange. The models also cannot take into account expectations about the future viability of the government. Their strict assumptions about taxation do not allow these models to account for tax deferment, tax evasion, tax avoidance, and tax exemption. Does the theory still hold if some people do not pay taxes some of the time?

In order to address these concerns I use for the first time a dynamic model to analyze the tax-foundation theory. Specifically, I use a monetary search model. In this class of models the monetary object is received in one period and later spent in shopping. Thus, I show that money not only has value but also functions as a genuine medium of exchange. I show that the theory holds as long as the government is expected not only to survive but also to maintain an effective tax-collection system. At the same time, the theory is sustainable even if taxes are not paid by everyone in every period. I also explore whether the tax-foundation theory is equivalent to convertibility or to government sales.[4] While there are strong similarities, one exceptional difference stands out: with this mechanism, the government can keep its money circulating while excessively printing more of it. This can explain the existence of numerous hyperinflations—where the money lost value but kept circulating while raising seigniorage revenue.

The mechanism explored here should not be confused with similar ones. It is an ingredient of Knapp's (1905) total state/chartal theory of money, according to which the government must support the money in many additional ways.[5] The tax-backing theory (Wallace 1981, Sargent 1982, Smith 1985) and the fiscal theory of the price level focus on the determination of the price level *given* that the money does circulate. In contrast, the present issue is whether any monetary equilibrium is realized *at all*, and the key determinant of that is not the tax rate or deficits, but rather which object is used to pay taxes, and what the penalty is for paying taxes with other objects. To emphasize the different focus of the tax-foundation theory, in my model public deficits can never exist. On the surface, the tax-foundation theory is similar to the legal restrictions theory (Cowen and Kroszner 1994, pp. 148-9). Both theories claim that inconvertible, intrinsically useless money may be valued only because of government regulation. Yet the two theories differ on an important matter: the legal restrictions theory claims that only the government is strong enough to suppress market-created money. The tax-foundation theory can imply that markets are too weak to create their own inconvertible, intrinsically useless money.

[4] Equivalence to government sales is implied in the Introduction of Li and Wright (1998).

[5] These include: accepting the money for any other payments such as fines, fees, payments for government-produced goods and services, and payments of banks to the central bank; using the money in its purchases; declaring it legal tender in private contractual debts; and fixing the money's exchange rate with the previous domestic money.

The paper is organized as follows. Section 2 explains in simple terms what "legal tender" really means. Section 3 presents the basic model with limited randomness in matching, exogenous prices, a unit upper bound on money holdings, and a fixed money supply. Section 4 introduces competing outside money and inside money. Section 5 allows multiple money holdings. Section 6 features endogenous prices and an increasing money supply. Section 7 relates the models to monetary history. I conclude in Section 8.

2 Legal Tender

The legal foundation of the monetary system is the law of legal tender. Monetary theorists have recently started labeling the money in their mathematical models as "legal tender." The meaning of this concept changes from paper to paper. It has been used to describe the only money that sellers are allowed to accept or the only money that buyers are allowed to offer. It has been used to describe money that buyers can force sellers to accept or money that sellers can force buyers to offer.[6] All these papers are wrong or misleading in relating the money they model to legal tender. Applying their results to real legal tender currencies can therefore be misleading. Some textbooks have similar problems, if they mention legal tender at all.[7] In fact, the legal tender concept has *nothing* to do with the spot transactions that dominate standard monetary theory. Its practical importance for circulation of money, if there is one, comes from its implication on tax payments.

So what is Legal Tender? According to United States Code 31 § 5103, "United States coins and currency (including Federal Reserve notes and circulating notes of Federal Reserve Banks and national banks) are legal tender for all debts, public charges, taxes, and dues." Legal tender applies only to situations in which there is disagreement on a medium of payment *after* a monetary obligation was created. Monetary obligations include contractual debts, tax payments, parking tickets, alimony, etc. Suppose that in one of these cases the debtor offers payment in some object but the creditor rejects it.[8] Can the creditor sue or prosecute the debtor in court for not paying the debt? Can the creditor insist on payment in an object which is exceedingly rare?

"Legal tender" is an object that confers a right on the debtor. If the debtor offers the correct quantity of anything that has been declared by law to be legal tender, then the creditor's lawsuit fails. The debtor

[6] See Lotz (2004), p. 967, Selgin (2003), p. 160, Shy and Tarkka (2002), pp. 303, 308, Lotz and Rocheteau (2002), p. 568, Sargent and Velde (2002), p. 368.

[7] See Lipsey, Courant and Ragan (1999), p. 581. Barro (1993, p. 96), mentions "legal tender" but does not explain what it means. Case and Fair (2003, p. 481), exclude tax payments from the legal tender law. Mankiw (2000, p. 156), only mentions an unspecified "government decree." Also see Auerbach and Kotlikoff, 1998, p. 175.

[8] The word "debt" in most legal tender laws includes any tax, while "creditor" includes any tax-collecting agency (Nussbaum 1950, pp. 49, 58, 139, Mann 1982, pp. 52, 80-100, European Union 1998, 2005, especially Articles 1, 8). The U.S. law distinguishes between debts and taxes but Federal Reserve notes only mention "debts." The latter is a relic from a time when courts interpreted "debt" as any obligation (United States, 2003, 31 § 5103, p. 21). Nobody bothered fixing it because it is the law that matters.

may be asked to deliver the proffered payment to court, which the court would offer to the creditor. The debtor is then off the hook, having fully performed the obligation of making payment.[9] On the other hand, any object that is not legal tender will not give the debtor such peace of mind. For this very practical purpose of settling disputes, every country specifies which objects are considered legal tender for debts that are subject to its contract law, for its tax obligations, and for other monetary obligations. Typically, the government gives this status to currency it issues itself, but this is not necessary.

Legal tender laws do not apply to spot transactions because in such transactions a disagreement on the medium of payment prevents the contract from coming into existence. Legal tender laws can apply in trade only if the parties agreed on some necessary terms that together constituted a legally valid contract (e.g., quantity, price and place of delivery), failed to mention the medium of payment to be given in the future, and *later* disagreed about what the medium of payment should be. If they do agree on the medium of payment, in the contract or after the contract is created, legal tender laws are irrelevant.[10] Almost any medium of payment on which the parties mutually agree is acceptable.[11] This is just one aspect of the freedom of contracts, which is a fundamental building block of capitalism. Legislatures have outlawed very few media of payment, such as gold (in post Great Depression legislation) or illegal drugs (which could conflict with the public interest). Another source of irrelevance of legal tender laws is that they apply only to obligations that are denominated in the official domestic unit of account. The legal tender law of the United States, which gives legal tender status to dollars in the form of coins and bills, cannot apply to contracts that specify payments in pesos or potatoes.

It turns out that sellers are not really forced to accept legal tender money if they are slightly cautious. They can insist on a spot transaction. Otherwise, they can insist that a different medium of payment be specified in the contract (and not sign on it otherwise). They can use a different unit of account. The websites of some central banks are honest about this limited legal status of their money.[12] The role of the state, after declaring what is legal tender, can be described as passive and negative: to dismiss a creditor's lawsuit if the debtor offers the right quantity of legal tender. A legal tender law never results in the state affirmatively prosecuting a buyer or a seller for using another currency or for rejecting the legal tender in a spot transaction. Other laws might do that, but they mostly exist during wars, in dictatorships and under strict socialism (Goldberg 2011).

In contrast, where the obligation is a tax payment the possibilities of a "spot" interaction or a different unit of account do not exist. Opportunities for agreeing on a medium of payment which differs from dollar

[9] Williston (2003), vol. 28, pp. 746, 805-14, Bank of England (2011), Reserve Bank of Australia (2011).

[10] Board of Governors of the Federal Reserve System (2011), Reserve Bank of Australia (2011).

[11] Bank of England (2011), Williston (2003), vol. 28, pp. 752-3, 778.

[12] E.g., Board of Governors of the Federal Reserve System (2011), Bank of England (2011), Bank of Canada (2011), Reserve Bank of Australia (2011).

notes and coins are limited and tend to be general for all taxpayers. Thus, checks and credit cards are accepted by the Internal Revenue Service not because the IRS must accept them but because it chooses to. The IRS actually encourages this explicitly by stating on tax forms: "Do not send cash." This rude order could not contradict the legal tender law more clearly, but virtually all taxpayers obey this request anyway for their own benefit (if you send your tax payment in cash, the mailman might steal it without a trace). For the IRS, this order is a weakly dominant strategy: it benefits the IRS if taxpayers obey, and in the unlikely case that one taxpayer does send cash, the IRS can simply accept it quietly upon arrival, with no harm done. The IRS can also choose to accept a tax payment from Boeing in the form of a new Air Force One, but neither side can force such a deal on the other. Such bilateral arrangements are less likely with the average taxpayer, and indeed the IRS rejected a tax payment in the form of a used car.[13] Generally, tax authorities accept only cash or financial instruments that are redeemable in this cash, like checks and credit cards.

How does all this relate to the circulation of money? In principle, legal tender laws can create demand by debtors for the objects which are legal tender. This demand can give value to these objects and contribute towards their general circulation. However, since it is so easy to avoid the applicability of legal tender laws in trade, this is not so trivial. Without calling it legal tender, Freeman (1996) showed in a model that if sellers believe that the money they cannot reject in debt payments will not circulate any further, they will simply choose not to be parties to any contracts. This can be overcome temporarily by applying a legal tender law retroactively to pre-existing contracts. This happened with the Civil War greenbacks, and invoked a constitutional firestorm. Creditors who did not expect this first U.S. paper money failed to specify "gold" in their contracts. Debtors were happy to pay debts with paper greenbacks instead of gold, and their demand for the greenbacks gave the greenbacks value. More important is the role of taxes because they are not voluntary to begin with for either the tax authority or the taxpayer. The rest of this chapter therefore focuses on taxes.

3 The Basic Model

One goal of this chapter is to model the tax-foundation theory in a monetary search model which is currently the workhorse of mathematical theories of money. Such models typically exhibit complete randomness of all meetings between agents; therefore they are also known as random matching models. However, a salient feature of real-life taxation is some lack of randomness. People know when they are going to be taxed and they usually know the terms: how much, where, how and in what medium of payment. They also have enough time to prepare for a tax payment. Some taxes can be completely, and legally, avoided by eschewing certain activities. It is thus useful to have a model in which taxation

[13] United States (2003), 31 § 5103, p. 27, note 17.

is not completely random. The alternative of complete randomness, which is less realistic but closer to standard models, is featured in the working paper (Goldberg 2010). No result whatsoever is driven by this or that assumption about matching.

As for the trade meetings, I maintain the same randomness as in random matching models. I do this for the sake of comparison with random matching models of competing mechanisms. In particular, the model is designed to be as close as possible to Li and Wright (1998), who model sales of government goods for money as a way to support that money.

3.1 Environment

Time is continuous. A continuum $[0,1]$ of infinitely-lived agents are randomly matched in pairs according to a Poisson arrival rate α. A fraction $p \in (0,1)$ of them are private agents. Each private agent derives utility $U > 0$ from the consumption of one indivisible unit of some goods. The agent can produce only one type of good, which it does not consume. Production of one unit of any (perishable) good is instantaneous and involves disutility $C \in (0,U)$. Production is independent of previous consumption, but agents can produce if and only if they do not hold money.[14] The probability that any private agent consumes the good of its trading partner is $x > 0$, and the conditional probability that the converse holds too is $y \geq 0$. As in Lagos and Wright (1998), αx is normalized to 1. The discount rate is $r > 0$.

At present, the only durable objects are dollar bills, which have a storage cost $c > 0$ and a fixed supply $M_\$ \in [0, \min(1-p, p))$.[15] A fraction m of private agents are initially endowed with one dollar each and are called *private buyers*. The other private agents hold nothing and are called *sellers*.

The rest of the randomly matched agents are *potential government buyers*. They can neither produce nor store anything. This assumption, starkly different from Aiyagari and Wallace (1997) and Li and Wright (1998), emphasizes that the tax-foundation mechanism, unlike convertibility, can work even for a government which is completely parasitic and cannot credibly promise convertibility into real goods.

Some of the potential government buyers hold one bill each. Such a *government buyer* receives a taste shock when it meets a seller. With probability x it wants to consume the seller's good. It offers the bill but does not force the seller to trade. If the money is accepted, the government buyer consumes and becomes a potential government buyer. Having no money, it can do nothing until it receives a new bill, as described below.

In addition to all these randomly matched agents, there is another class of government agents, called *tax collectors*. They operate outside of the matching process described above. They are idle during trade

[14] This assumption differs from Li and Wright (1998). It allows agents who were just taxed to resume market activities, while keeping one unit of money as an effective upper bound on their holdings.

[15] The assumption $M_\$ < 1-p$ will allow the government to hold all the money if private agents reject it (as in Aiyagari and Wallace 1997). The assumption $M_\$ < p$ will be useful in establishing a monetary equilibrium.

meetings and operate right after them (as in Shi 2005). The tax collectors are capable of identifying agents who had just produced. Tax law in the economy states the following: 1. Only current income is taxed. 2. An agent who just sold is taxed with probability $\tau \in [0, 1)$. 3. The size of the tax payment is the entire new income. 4. Tax collectors cannot reject tax payments in dollar bills. 5. Tax collectors can choose whether to accept a tax payment in a real good. 6. Agents whose payment is rejected face a non-monetary punishment $P > 0$.[16]

This tax law is different from totally random taxation. Agents know that only a sale makes them eligible for paying the tax. They can choose never to be exposed to the tax. They can also choose to make a sale only if it leaves them well prepared for the tax collector's visit (say, sell for bills but avoid barter). Nevertheless, some randomness ($\tau < 1$) must remain for the sake of analytical tractability.[17] This limited randomness is also useful in approximating tax evasion, tax avoidance, tax exemption, tax deferment and the fact that income taxes are not paid after every single sale. The earlier models of the tax-foundation theory already have $\tau = 1$ (Starr 1974, 2003). Let G denote the subjective probability that private agents assign to the existence of a taxing government in the next period. Denote the expected probability of being taxed as $t \equiv \tau G$.

Sections 4 and 5 of the tax law specified above are a real-life legal tender law. Explicitly, the law imposes an obligation only on tax collectors, and it says nothing about objects other than dollar bills. However, its silence regarding other objects means – according to basic legal principles – that tax collectors have full discretion whether to accept such objects.

After taxes are collected all the proceeds are transferred to the potential government buyers, with each buyer getting one bill at most. Then trade meetings resume. Note that the government always has a balanced budget in the sense that taxation precedes consumption for all government buyers.

3.2 Strategies

Sellers can barter in some cases and sell for bills in other cases. Let Π_0 be the probability that a random seller agrees to barter. It is not trivially equal to 1 as in standard models because barter makes a seller eligible for taxation. Let $\Pi_\$$ be the probability that a random seller accepts a bill. Let the probabilities π_0 and $\pi_\$$ be the stationary best responses of a maximizing seller who is offered a good it consumes, and a bill, respectively. Let T_0 indicate whether tax collectors accept real goods ($T_0 = 1$) or not ($T_0 = 0$). If

[16] The punishment can be thought of as beating. It is possible to model it as a fine paid in real goods produced by the offender, as in Soller-Curtis and Waller (2000).

[17] If $\tau = 1$ was allowed, the only way to keep production incentive-compatible would have been to tax only a fraction of one's income. This would have necessitated an increase in the upper bound on the indivisible money holdings and would have precluded closed-form solutions.

V_0 and $V_\$$ are the value functions of sellers and private buyers respectively, then

$$rV_0 = p(1-m)y\Pi_0 \cdot \max_{\pi_0}\pi_0\{U - C + t[T_0(-C) + (1-T_0)(-P)]\} + M_\$ \cdot \max_{\pi_\$}\pi_\$[-C + (1-t)(V_\$ - V_0)], \quad (1)$$

$$rV_\$ = -c + p(1-m)\Pi_\$(U + V_0 - V_\$). \quad (2)$$

In (1) a seller has two interesting matching possibilities. It may have double coincidence of wants with another seller. With probability t a bartering agent later meets a tax collector. If the tax collector agrees to accept the agent's produce then the seller produces. Otherwise, the seller faces the punishment P. Note that choosing $\pi_0 = 0$ ensures non-exposure to P. If the seller meets any buyer (public or private), it chooses whether to accept a bill. If it does, then it produces and gets \$1. Again, this makes the seller eligible for taxation. With probability t it is taxed of all its money so it remains a seller. With probability $1-t$ the agent is not taxed so it becomes a buyer. Note that because agents can choose $\pi_0 = \pi_\$ = 0$, we have $V_0 \geq 0$. No matter how large P is, agents can choose not to be exposed to the punishment. In (2) a buyer pays the storage cost of the bill and it can buy a good if the seller it meets accepts a bill.

3.3 Equilibrium

The analysis is restricted to symmetric, pure strategy, *non-autarkic* equilibria. Three definitions will be useful. In a *pure monetary equilibrium* all sellers accept bills but do not barter. In a *monetary equilibrium* all sellers accept bills and barter. In a *non-monetary equilibrium* all sellers barter but reject bills.

At the heart of the model stand two decisions. First, the tax collectors decide which objects to receive in tax payments in addition to dollar bills. Second, the legislature determines the punishment for those whose tax payment is rejected.

Definition 1. An object i, $i \in \{0, \$\}$, is *tax-receivable* iff $T_i = 1$.

The tax law already stated that $T_\$ = 1$.

It is easy to discourage barter (and production in general), by simply setting a high enough probability of taxation. The point of this chapter, however, is not to describe such a mechanism, but to relate to the discrimination regarding tax-receivable objects. The focus here is not on the probability of payment but rather the *medium* of payment. Assumption 1 below ensures that without such discrimination, barter — and therefore the non-monetary equilibrium — can exist.

Assumption 1. $t < U/C - 1$.

The first result is that if all objects are tax-receivable, the bills may not circulate at all.

Proposition 1. If all objects are tax-receivable, then: (i) the non-monetary equilibrium exists; (ii) the monetary equilibrium exists for some parameter values; (iii) a pure monetary equilibrium does not

exist.

Proof. (i) Set $T_0 = \Pi_0 = 1$ and $\Pi_\$ = 0$ in (1) and (2). Clearly, $\pi_\$ = 0$, while Assumption 1 implies $\pi_0 = 1$. With barter and rejection of bills, we have the non-monetary equilibrium. (ii) Set $T_0 = \Pi_0 = \Pi_\$ = 1$. Then $-C + (1-t)(V_\$ - V_0) > 0$ (so $\pi_\$ = 1$) iff $c < c_e$, where $c_e \equiv [p - M_\$(1-t)](1-y)U + \left\{[p - M_\$(1-t)]y(1+t) - \frac{r+p-M_\$(1-t)}{1-t}\right\} C$. In this equilibrium $m = M_\$(1-t)/p$. (iii) Assumption 1 implies $\pi_0 = 1$. □

As usual in monetary models, the circulation of an object which is intrinsically useless and not convertible depends on both its intrinsic properties and agents' beliefs. It will not circulate if c is high.

3.4 Policy

The government may be able to affect existence of equilibria by discriminating between various objects. Specifically, the government can make the dollar bill the *only* tax-receivable object. It then needs to determine the punishment for those not paying taxes in bills.

Theorem 1. The government can guarantee the *existence* and *uniqueness* of the pure monetary equilibrium iff $t > 0$ and c is small enough.

Proof. Set $T_0 = 0$ and $P > (U - C)/t$. This implies $\pi_0 = 0$ so there is no barter. This rules out the non-monetary and monetary equilibria for all parameter values. Setting $\Pi_\$ = 1$ implies that $\pi_\$ = 1$ iff $c < c_u$, where $c_u \equiv [p - M_\$(1-t)]U - \frac{r+p-M_\$(1-t)}{1-t} C$. □

Without the policy, bills may circulate only if $c < c_e$. With policy, bills circulates for sure iff $c < c_u$. It is always the case that $c_e < c_u$, which means that in some cases policy enables the bills to circulate when it otherwise would not. Looking at P in the proof, it is clear that the lower the probability of meeting tax collectors in the future, the higher the minimal punishment needs to be.[18] The mechanism does not work if agents expect the tax collectors to disappear soon ($t = 0$), either because the government is expected to disappear ($G = 0$) or because its tax system is expected to disappear ($\tau = 0$). This has important implications on the historical relevance of the theory, as elaborated in Section 7.

By rejecting any other object in tax payments, the government artificially creates a demand for the legal tender objects and makes them valuable (Smith 1776). Taxpayers must obtain legal tender objects in order to pay their taxes. While denominating a contract in foreign currency makes the legal tender law irrelevant for your contract, receiving your entire income in foreign currency will not exempt you from paying your taxes in dollars. Taxpayers are therefore willing to provide goods and services for the legal tender objects. This can result in their circulation as media of exchange (Lerner 1947).

[18] I vary only the punishment and keep the tax rate fixed, since tax rates today are usually determined by the fiscal needs of the government, rather than by the need to support monetary equilibria. It is the insistence that these taxes be paid in a certain money, and the associated punishment, that can serve monetary equilibria as a positive externality. In colonial America many tax rates were determined so as to deliberately support monetary equilibria (Brock 1975).

It is important to compare the tax-foundation mechanism to convertibility. The latter is a commitment of the issuer to convert paper money into gold or any other good or service. In the tax-foundation mechanism the government does not give anything useful for paper money. However, it does give something *harmful* if one does *not* have paper money when it is time to pay taxes. Paper money thus buys immunity from punishment. It is implicitly convertible into immunity. The mechanism is therefore somewhat analogous to convertibility. However, it is not an equivalent. It involves the government by definition, because only the government taxes. Convertibility, on the other hand, has been practiced by both governments and a wide variety of private entities. Close to convertibility are models of the government as a seller of goods (Aiyagari and Wallace 1997, Li and Wright 1998). In both these models and the current one, more interaction with the government implies that more private agents face an exogenous pro-money behavior, which inspires general circulation of money. The difference is that here the government's crucial role is as a tax collector rather than a seller. In a modern economy the government sells very few goods in the marketplace, but its taxation is always present and is considerable.

In fact, this model does not critically depend on any government involvement in trade or monitoring of trade: the government does not convert bills into real goods, it does not directly force agents to use bills in their own trade (barter is legal), and it does not force sellers to accept bills from government buyers. Here, the only role that the government has in trade is a technical one: to return the money that it collects to the economy through its buyers, as real-life governments do. I could have assumed instead that the government destroys the collected money and then injects new money by helicopter drops.

Is the policy optimal?

Proposition 2. (i) The pure monetary equilibrium has lower welfare than the monetary equilibrium. (ii) If y is small enough, the pure monetary equilibrium is better than the non-monetary equilibrium.

The proof is trivial. There is a trade-off as in Aiyagari and Wallace (1997). In that model, the government supports money by refusing some barter opportunities. This means that some welfare is lost, compared to a monetary equilibrium without such policy. However, the resulting monetary equilibrium may be better than the non-monetary equilibrium that could have existed without the policy. The only difference is that the foregone barter opportunities in the current model are always between private agents. The policy induces them to give up those trades, but guarantees the use of money. Monetary trade is more likely to be the optimal form of trade if direct barter is difficult (i.e., y is small).

One more policy should be considered, because it clarifies the crux of the mechanism and because it relates to the historical discussion below. Suppose that the tax is denominated in goods rather than bills. In particular, it is set at one good per taxpayer. In addition to accepting goods, tax collectors also accept bills, but only according to their market value. If an agent pays in a bill whose market value is zero ($\Pi_\$ = 0$) then the tax collector forces this agent to produce another good as payment. A seller's

value function is now

$$rV_0 = p(1-m)y\Pi_0 \cdot \max_{\pi_0} \pi_0[U - C(1+t)] + M_\$ \cdot \max_{\pi_\$} \pi_\$[-C - t(1-\Pi_\$)C + (1-t)(V_\$ - V_0)]. \quad (3)$$

Proposition 3. A policy of accepting money for taxes at market value is ineffective.

Proof. $\pi_0 = 1$ because there is no punishment. For there to be any equilibrium at all, set $\Pi_0 = 1$ as well. Setting $\Pi_\$ = 0$ replicates the qualitative outcome of Proposition 1(i), while $\Pi_\$ = 1$ exactly replicates Proposition 1(ii). Proposition 1(iii) holds here too. □

The key point with regards to government intervention is that it introduces *exogenous* behavior, which is unrelated to endogenous market expectations. Once the policy itself succumbs to these expectations and accommodates them, it has no hope of any real impact.

4 Competing Currencies

In the real world dollar bills have competed not only with barter but mainly with other currencies, such as foreign currency, private banknotes and checks, and commodity money. Modelling commodity money is quite a challenge because there is both an intrinsic value (as in barter) and a potential to function as money (like foreign currency). This requires a very different model from the one used here. In Goldberg (2002) I solve numerically a model of that type, as a direct modification of the first monetary search model (Kiyotaki and Wright 1989). The results are weaker: circulation of bills can be guaranteed but commodity money cannot be excluded. That is, the government can guarantee a monetary equilibrium but not a pure monetary equilibrium. A brief sketch of the model is available in the working paper (Goldberg 2010).

Here I examine two other competing currencies, one at a time: outside money (say, euro, denoted e) and inside money (say, banknote, denoted b). Let M_i, V_i, T_i, π_i, and Π_i, $i \in \{0, \$, e, b\}$, be the obvious generalization of the above notation.

4.1 Another Outside Money

Suppose that euro bills have the same physical properties of dollar bills, including the storage cost c. To simplify the analysis, I assume that currency trading of dollars for euros is impossible. Due to the symmetry between agents and between currencies, such trading could not be mutually beneficial in equilibrium, and thus would not occur in any case. The value functions are now determined as follows.

$$rV_0 = p(1-m)y\Pi_0 \cdot \max_{\pi_0} \pi_0\{U - C + t[T_0(-C) + (1-T_0)(-P)]\} + M_\$ \cdot \max_{\pi_\$} \pi_\$[-C + (1-t)(V_\$ - V_0)] + \quad (4)$$

$$M_e \cdot \max_{\pi_e} \pi_e[-C + t(1 - T_e)(-P) + (1 - t)(V_e - V_0)],$$

$$rV_i = -c + p(1 - m)\Pi_i(U + V_0 - V_i), \tag{5}$$

for $i = \$, e$.

Proposition 4. If all objects are tax-receivable and $c < c_e$ there are four equilibria coexisting: a non-monetary equilibrium; a monetary equilibrium with the dollar as the unique money; a monetary equilibrium with the euro as the unique money; and a monetary equilibrium with both dollars and euros as money. There is no pure monetary equilibrium.

Proof. Essentially identical to the proof of Proposition 1.

Suppose that the government wants to promote the use of dollars as money.

Theorem 2. The government can guarantee the *existence* and *uniqueness* of the pure monetary equilibrium with dollars iff $t > 0$ and the bills' storage cost is small enough.

Proof. Set in (4) $T_0 = T_e = 0$. Setting $P > (U-C)/t$ rules out barter (and therefore all non-monetary and monetary equilibria). It is also sufficient for ruling out the pure monetary equilibrium in which the euro is the unique money. A possibly higher punishment, $P > \frac{[(1-t)U-C][p-(M_\$+M_e)(1-t)]-(1-t)c-rC}{t[r+p-M_e(1-t)]}$, is needed to rule out the pure monetary equilibrium in which both dollars and euros are money. This leaves only the pure monetary equilibrium with the dollar as the unique money. □

In general, if there are n types of intrinsically useless objects with the same low storage cost, then without government intervention there are $\sum_{x=1}^{n} \binom{n}{x}$ pure monetary equilibria and the same number of monetary equilibria. The non-monetary equilibrium also exists if the government does not intervene, so overall there are $2^{n+1} - 1$ equilibria. The government can make any of the n objects the unique money in a unique pure monetary equilibrium. Note that the other outside money modeled here could also be paper money from the game Monopoly or money of the late Ottoman Empire. In principle, they could also be used as money if everyone happens to believe that they will.

4.2 Inside Money

Instead of euros let us now assume that there is another entity in the economy called a bank. It has a fixed location which is costlessly accessible to all agents between trading and taxation. It has a unique technology which enables it to produce banknotes, which are durable, indivisible and are not consumed. Their advantage over dollar bills is that they have no storage cost. This advantage represents the convenience of banknotes compared with gold coins in the past, and the convenience of checks, debit cards and electronic money compared with dollar bills today.

An agent who holds a dollar bill after a trade round can go to the bank, deposit the dollar bill, and

receive a lighter banknote instead. In return for this service the agent has to produce for the bank.[19] The bank consumes all the real goods. Any agent holding a banknote can go to the bank after a trade round and try to convert it into a dollar bill. There is no cost to such conversion. The bank keeps a 100% reserve ratio but might vanish with probability $1-R$ after each trade round, where $R \in [0,1)$.[20] Therefore, conversion succeeds with probability R.

Let π^S_{ij}, for $i,j \in \{\$, b\}$, be the strategy of a seller who just earned some money i (and thus can be taxed) and chooses whether to convert it at the bank into money $j \neq i$. Let π^B_{ij} be the similar strategy of a buyer who had money i before the last trade round and failed to spend it. Due to the increased complexity of the model, it is useful to let $y = 0$ here, and to assume that buyers cannot swap a dollar and a banknote among themselves. These simplifications do not affect the results. The value functions are now

$$rV_0 = (M_\$ - M_b) \cdot \max_{\pi_\$, \pi^S_{\$b}} \pi_\$\{-C + (1-t)(V_\$ - V_0) + \pi^S_{\$b}[-C + t(1-T_b)(-P) + (1-t)(V_b - V_\$)]\} + \quad (6)$$

$$M_b \cdot \max_{\pi_b, \pi^S_{b\$}} \pi_b\{-C + \pi^S_{b\$}R(1-t)(V_\$ - V_0) + (1-\pi^S_{b\$}R)[t(1-T_b)(-P) + (1-t)(V_b - V_0)]\},$$

$$rV_\$ = -c + p(1-m)\Pi_\$(U + V_0 - V_\$) + [1 - p(1-m)\Pi_\$] \cdot \max_{\pi^B_{\$b}} \pi^B_{\$b}(-C + V_b - V_\$), \quad (7)$$

$$rV_b = p(1-m)\Pi_b(U + V_0 - V_b) + [1 - p(1-m)\Pi_b] \cdot \max_{\pi^B_{b\$}} \pi^B_{b\$}R(V_\$ - V_b). \quad (8)$$

In (6) the probability of meeting a bill holder depends on how many bills are stored at the bank (i.e., how many banknotes are outstanding). A seller who just earned a dollar can exchange it for a banknote by producing for the bank. If it is later taxed and the tax collector refuses to accept banknotes then it is punished. If it accepts a banknote in trade (second line of (6)) it can try to redeem it at the bank for a dollar bill. The motivation for this is the possibility that banknotes are not tax-receivable. Not only a recent seller can deposit a bill at the bank: in (7) a buyer who fails to buy with a bill can deposit it. In (8), a buyer holding a banknote can try to convert it into a dollar. Since barter is ruled out by assumption, only pure monetary equilibria are possible.

Proposition 5. If all objects are tax-receivable, then: (i) the pure monetary equilibrium with dollar

[19] Technically, the seller can produce only when it holds nothing, i.e., after it gives the dollar to the bank and before receiving the banknote.

[20] The assumption $R < 1$ is critical. If the costless redemption is also riskless, then agents never reject banknotes even if banknotes are not tax-receivable. The reason is timing: agents can go to the bank between trade and taxation and convert a banknote into a tax-receivable dollar bill.

bills exists for some parameter values. (ii) for a different set of parameter values there is a pure monetary equilibrium in which banknotes circulate and are used in tax payments, while all the dollar bills — although not rejected in trade — are actually always at the bank. (iii) there can be a pure monetary equilibrium with banknotes in which dollar bills are rejected.

Proof. (i) Set in (6)-(8) $T_b = \Pi_\$ = 1$, $\Pi_b = 0$. Banknotes are rejected in trade ($\pi_b = 0$) iff $c \leq [p - M_\$(1-t)]U - \frac{(R+r)(r+p)M_\$ C}{[r+(1-t)M_\$][(1-t)(1-R)M_\$ - R(1+r)]}$ and $R < \frac{(1-t)M_\$}{(1-t)M_\$ + 1 + r}$. It is optimal to convert banknotes into bills ($\pi^S_{b\$} = \pi^B_{b\$} = 1$) iff $c < [p - M_\$(1-t)]\left[U - \frac{M_\$ C}{r+(1-t)M_\$}\right]$. This condition is sufficient to prevent depositing of bills ($\pi^S_{\$b} = \pi^B_{\$b} = 0$). The rest is exactly as in Proposition 1(ii). (ii) Set $T_b = \Pi_\$ = \Pi_b = 1$. Banknotes are accepted in trade iff $[p - M_\$(1-t)][(1-t)U - C] > rC$.[21] Dollar bills are deposited by sellers iff $c > \left[\frac{r+t}{1-t} - p + M_\$(1-t)\right] C$. This condition is sufficient for a buyer with a dollar to deposit it. It is not optimal to convert banknotes into dollars. (iii) With dollars rejected in trade, equations (6) and (8) are essentially the same as (1) and (2), only that there is no barter, there is no storage cost, and the notation "b" replaces "\$" everywhere. Thus Proposition 1 applies. □

Theorem 3. The government can guarantee the existence and uniqueness of the pure monetary equilibrium with dollar bills iff $t > 0$ and c is small enough.

Proof. Set $T_b = 0$ and $\Pi_\$ = 1$. In (6), regardless of the value of Π_b, a high enough P results in rejection of banknotes and no depositing of dollars. The rest is as in Theorems 1 and 2. □

As with any piece of paper, the circulation of a banknote or a check depends on agents' beliefs. The tax law gives tax collectors full discretion whether to accept banknotes. If they choose to reject them, they can drive the banknotes out of circulation, as in Theorem 3. However, they can also choose to accept them and not disrupt this efficient use of a lighter medium of exchange, as in Proposition 5(ii).

5 Multiple Money Holdings

An obvious shortcoming of the model is that agents never hold more than one object at a time. One might suspect that allowing more flexibility would allow agents to diversify their portfolios or at least be flexible about what they accept in payment. Assume then that agents can produce only if they hold up to one object of any type. This effectively increases the upper bound on money holdings from one unit to two units. Prices are still fixed for now at 1.

The point can be made by considering the economy with barter and only dollar bills as potential money. Let m_i, $i \in \{0, \$, \$2\}$, be the fraction of private agents holding i. For $i, j \in \{0, \$\}$, let Π_{ij} be the probability that a random seller holding i accepts j, and let π_{ij} be the best response of an agent who

[21] This is the same condition from Proposition 1(ii), only here $y = 0$ and banknotes have no storage cost.

holds i and is offered j. Let $V_{\$2}$ be the value function of an agent holding \$2. The value functions are

$$rV_0 = p(m_0\Pi_{00} + m_{\$}\Pi_{\$0})y \cdot \max_{\pi_{00}} \pi_{00}\{U - C + t[T_0(-C) + (1 - T_0)(-P)]\} + \tag{9}$$

$$[M_{\$} - pm_{\$2}] \cdot \max_{\pi_{0\$}} \pi_{0\$}[-C + (1 - t)(V_{\$} - V_0)],$$

$$rV_{\$} = -c + p(m_0\Pi_{00} + m_{\$}\Pi_{\$0})y \cdot \max_{\pi_{\$0}} \pi_{\$0}[U - C + t(V_0 - V_{\$})] + \tag{10}$$

$$[M_{\$} - pm_{\$2}] \cdot \max_{\pi_{\$\$}} \pi_{\$\$}[-C + (1 - t)(V_{\$2} - V_{\$})] +$$

$$p(m_0\Pi_{0\$} + m_{\$}\Pi_{\$\$})(U + V_0 - V_{\$}),$$

$$rV_{\$2} = -2c + p(m_0\Pi_{0\$} + m_{\$}\Pi_{\$\$})(U + V_{\$} - V_{\$2}). \tag{11}$$

In the first line of (9) a seller meets a barter partner who has either \$0 or \$1. The second line is a meeting with a buyer who has at least \$1. In (10) an agent with \$1 has three interesting matches. First, it may meet a barter partner. In this case, if it makes a sale it may be taxed and then it ends up with no money. Second, it may make a monetary sale and accumulate another dollar. Third, it may spend the money in shopping. In (11) the agent suffers the storage cost twice because it holds two dollar bills. It can only buy.

Proposition 6. (i) The government can guarantee that the non-monetary equilibrium does not exist iff $t > 0$. (ii) For some parameter values the government can guarantee the existence of a monetary equilibrium. (iii) The government cannot guarantee the existence of a pure monetary equilibrium.

Proof. (i) Set $T_0 = 0$. By setting $P > (U - C)/t$, moneyless agents will not barter ($\pi_{00} = 0$). (ii) Also set $\Pi_{00} = 0$, $\Pi_{\$0} = \Pi_{0\$} = \Pi_{\$\$} = 1$. There is no closed-form solution because the distribution of money is too complicated. It can be verified that $\pi_{\$0} = \pi_{0\$} = \pi_{\$\$} = 1$ for the following parameter values: $U = 5$, $C = 1$, $M_{\$} = .5$, $r = .01$, $p = .9$, $t = .25$, $y = .3$, $c = .01$. This means that agents always accept bills, but they barter only if they already have bills. (iii) No matter how high the punishment is, an agent who already has \$1 may still barter. The reason is apparent from (10): such an agent barters ($\pi_{\$0} = 1$) iff $U - C + t(V_0 - V_{\$}) > 0$. It cannot be punished. Although it does not earn money during this barter sale, it already has a dollar bill to begin with, so it can use that bill to pay the tax. □

These results obviously generalize to any larger upper bound on money holdings. For the same reason, the government cannot guarantee equilibria without other outside or inside moneys. An agent

who already holds a dollar bill may trade in any other way and pay that bill as a tax. On the other hand, the government can make sure that at least the moneyless agents accept dollar bills in sales. The result is that the government's favorite money still circulates, but not exclusively.

6 Endogenous Prices

Following Li and Wright (1998), I proceed by making goods divisible, while keeping everything else as in Section 2. Along with checking the robustness of the previous results, this also serves to show the robustness of the tax-foundation mechanism to money printing.

6.1 Robustness of Results

As in Li and Wright (1998) and similar papers that follow Trejos and Wright (1995), a private agent now derives utility $u(q) \geq 0$ from consuming a quantity q of one of its preferred goods, and the cost of producing a quantity q is q. Also $u(0) = 0$, $u'(q) > 0$ and $u''(q) < 0$ for all $q > 0$. In meetings between sellers the bargaining power is equal, so both sides produce the efficient q^* which satisfies $u'(q^*) = 1$. The quantity that may be produced for the tax collector after barter is also assumed to be q^*. In meetings between buyers and sellers, the buyers (whether private or public) make take-it-or-leave-it offers. The quantity produced in all other matches is denoted Q. Multiple money holdings is allowed only in the next subsection. The value functions are now

$$rV_0 = p(1-m)y\Pi_0 \cdot \max_{\pi_0} \pi_0\{u(q^*)-q^*+t[T_0(-q^*)+(1-T_0)(-P)]\}+M_\$ \cdot \max_{\pi_\$} \pi_\$[-q+(1-t)(V_\$-V_0)], \quad (12)$$

$$rV_\$ = -c + p(1-m)\Pi_\$[u(Q) + V_0 - V_\$]. \quad (13)$$

The analogous to Assumption 1 is the following.

Assumption 2. $t < u(q^*)/q^* - 1$.

Proposition 7. If all objects are tax-receivable, then: (i) the non-monetary equilibrium always exists. (ii) there are two monetary equilibria, which exist only for some parameter values. (iii) there is no pure monetary equilibrium.

Proof. (i) and (iii) are as in Proposition 1. (ii) Set $T_0 = \Pi_0 = \Pi_\$ = 1$. The bargaining rule implies $q = (1-t)(V_\$ - V_0)$, so $q = (1-t)\frac{[p-M_\$(1-t)]\{u(q)-y[u(q^*)-q^*]\}-c}{r+[p-M_\$(1-t)](1-yt)}$. As in Li and Wright (1998), the concavity of the utility function results in two solutions. $\pi_\$ = 1$ iff $q > 0$. □

The uniqueness result of Theorem 1 is replaced by something weaker.

Theorem 4. The government can guarantee the existence of two pure monetary equilibria, and that no other equilibria exist, iff $t > 0$ and c is small enough.

Proof. Setting $P > [u(q^*) - q^*]/t$ and $T_0 = 0$ eliminates barter. Setting $\Pi_\$ = 1$ results in $q = (1-t)\frac{[p - M_\$(1-t)]u(q) - c}{r + p - M_\$(1-t)}$. $\pi_\$ = 1$ iff $q > 0$. $\square$

6.2 Money Printing

I will now show that the government can maintain the circulation of its favorite money while increasing its supply and decreasing its value. Consider an economy identical to the one above, only that the money supply is $XM_\$$ one-dollar bills and each buyer is endowed with X one-dollar bills, for $X \in \{2, 3, 4, ...\}$.[22] This economy can be thought of as the result of an unexpected once and for all transfer of $\$(X-1)$ to each buyer in the abovementioned economy. For $i = 1, 2, 3, ..., X$, let b_i be the probability that a seller meets a buyer who offers $\$i$, let q_i be the quantity demanded by such a buyer, let Q_i be the quantity produced in all other matches for $\$i$, let Π_i be the probability that a random seller accepts a payment i, let π_i be the best response of a seller who is offered a payment i, and let V_i be the value function of an agent holding $\$i$. Then

$$rV_0 = p(1-m)y\Pi_0 \cdot \max_{\pi_0} \pi_0 \{u(q^*) - q^* + t[T_0(-q^*) + (1-T_0)(-P)]\} + \tag{14}$$

$$\sum_{i=1}^{X} b_i \cdot \max_{\pi_i} \pi_i [-q_i + (1-t)(V_i - V_0)],$$

$$rV_i = -ic + p(1-m)\max\{\Pi_k[u(Q_k) + V_{i-k} - V_i]\}_{k=1}^{i}, \tag{15}$$

for $i = 1, 2, 3, ..., X$.

In (14) a seller can meet buyers who hold up to $\$X$. Each one of them has a favorite offer of $\$i$. If taxed, the seller still pays the entire income, in accordance with the tax law. In (15) the buyer might meet different types of sellers. Some might accept only small payments (and provide a small quantity of goods) because of the bills' storage cost. Others might accept only large payments (e.g., if all prices are even it is useless to hold a one-dollar bill). Subject to the sellers' willingness, the buyer chooses how much to pay.

Theorem 5. The government can still guarantee the existence of two pure monetary equilibria, and that no other equilibria exist, iff $t > 0$ and c is small enough.

Proof. Setting $P > [u(q^*) - q^*]/t$ and $T_0 = 0$ eliminates barter. Setting $\Pi_i = 0 \ \forall i = 1, 2, ..., X-1$, and $\Pi_X = 1$, results in $\pi_i = 0 \ \forall i = 1, 2, ..., X-1$, and $q = (1-t)\frac{[p - M_\$(1-t)]u(q) - Xc}{r + p - M_\$(1-t)}$. $\pi_X = 1$ iff

[22] Recall that throughout this chapter agents are not physically prevented from holding more than \$1 at a time. Only a restriction on their production capabilities prevents such accumulation in most of the chapter. I maintain the assumption (relaxed in the previous section) that only moneyless agents can produce.

$q > 0$. $\square$

The only difference from Theorem 4 is that c is multiplied by X. Buyers spend X times more money in each trade. If c is very small (as in real-life paper money) and X is not too large (think of anything other than Germany in 1923), then the extra storage cost affects bargaining only slightly. Output barely changes and prices are almost exactly X times their former value. Money is almost neutral. Even these minor real changes are gone if the government replaces every bundle of X one-dollar bills with a single X-dollar bill.

Guaranteeing the circulation of money is therefore just as easy with more money than with less money. A proportional monetary transfer is neutral not only in terms of allocations, but also in the strength of the pure monetary equilibria and the policy's viability. In contrast, excessive printing of convertible money (where each bill is a promise to a gold coin), might lead to collapse because of loss of trust that the promise could be fulfilled. The tax-foundation mechanism is similar to convertibility in its ability to guarantee *circulation*, under certain conditions, but it cannot guarantee the *value* of money because it allows a discretionary money supply (Woodward 1995, p. 929). The government can print money at will, causing higher prices, higher nominal incomes — and thus *higher nominal tax obligations*. The price of immunity from being punished as a tax offender is indexed to the money supply, and so the money's acceptability is sustained. While some see this "indeterminacy" of the price level as a fatal flaw for a theory of money (Ellis 1934, Klein 1974), governments have realized its immense power: they can guarantee that their money circulates (as with convertibility), even though they can print more of it at will. They can have their cake and eat it too. Therefore, this indeterminacy of the price level is a reason to understand the tax-foundation theory rather than ignore it.

Between convertibility and the tax-foundation mechanism there is the mechanism in which the government sells goods for its money. As in convertibility, the government provides real goods for money. As in the tax-foundation mechanism, the prices are not fixed but are indexed to the money supply. Money printing raises the prices of goods and thus agents need more money to pay for these goods. In theory we have the model of Li and Wright (1998). In history we have the sale of land for *assignat* paper money in the French Revolution. The weakness of this mechanism in France was that people did not have to buy land, and certainly did not have to buy more and more of it all the time. In contrast, paying taxes (i.e., avoiding imprisonment) is something that, generally speaking, all agents badly need, and they need it again and again in every period.

7 Monetary History

In this section I discuss the relevance of the model to reality. Parts of the tax law described in Section 3 are in the legal tender law of every modern country. Every such country declares at least one object

to be legal tender for taxes (and contractual debts). Usually only one class of such objects — coins and notes of different denominations issued by the domestic government — has this status. The tax-collection authority cannot reject tax payments made in the various objects that belong to this class of objects. It can reject anything else and it usually does.

Casual evidence is clearly favorable to the hypothesis that there is a relation between what is declared legal tender and what circulates as money. Given political and economic stability, indeed the domestic legal tender is typically the general medium of exchange. Countries and currency unions seem to be able to change currency at will, as recently seen in Iraq and the European Union. As Goodhart (1998) notes, money tends to follow political unification and disintegration of federations: German states adopted one money in the 19th century while former Soviet republics have their own moneys.

It is easy to find in the literature cases of moneys which failed even though they were receivable for taxes. The first task of this section is to show that such cases can be explained *within* the model and the theory outlined above. Some economists are quick to cite many cases of moneys that supposedly circulated without being receivable for taxes (and were also neither intrinsically valuable nor convertible). If true, this would mean that the tax-foundation mechanism is not necessary in practice and perhaps contributes nothing. The second task of this section is to show that there is very little substance in such evidence. Once the facts are examined carefully and the theory interpreted properly, by and large, both the successes and failures of intrinsically useless, inconvertible moneys in reality seem to be related to the tax-foundation mechanism.

7.1 Failures of the Tax-Foundation Mechanism

The model developed above is helpful in explaining why sometimes an object is receivable for taxes and yet does not circulate.

7.1.1 Non-Unique Legal Tender

The Civil War's greenbacks were rejected in favor of gold in the West Coast (Mitchell 1903) even though they were legal tender and thus acceptable in tax payments. This failure is consistent with the theory because gold was also legal tender. Proposition 1 shows that if something which is not paper money is receivable for tax payments, there is always an equilibrium in which paper money does not circulate. Taxpayers did not have to use greenbacks for tax payments. They could get along just fine with gold, and therefore sufficient demand for greenbacks could not be guaranteed.

More recently, the American public has rejected the half-dollar coin, the Susan B. Anthony dollar, and the two-dollar bill. All of these are also legal tender and therefore acceptable in tax payments, but again they are not the exclusive objects which are legal tender. According to Proposition 4 if two different objects have the same storage cost and both are receivable for taxes, then either one of them or both

may circulate. Since people do not have to pay taxes in coins and notes of particular denominations, the failure of some denominations does not contradict the theory. In both cases, then, the model features multiple equilibria. Indeed, other evidence shows the exact opposite behavior: during the Revolutionary War gold was hoarded while the paper continental was used in trade (Calomiris 1988); and the recent Sacagawea golden dollar coins were happily received by sellers but were hoarded instead of being spent (Lotz and Rocheteau 2002).

7.1.2 Regime Transition

The observed correlation between a paper money's circulation and the existence of its issuing regime is highly positive but not perfect. Saddam Hussein's money circulated in Iraq for half a year after his regime collapsed. On the other hand, Germans abandoned their paper money a few months before the Nazi surrender (Einzig 1966, p. 299). These observations are not inconsistent with the theory. All the propositions and theorems above that show the government's power to promote its favorite currency include the condition $t > 0$, where $t \equiv \tau G$ is the expected probability of being taxed in the future by a government that accepts this currency (recall that τ is the probability of meeting tax collectors and G is the government's survival probability). In such episodes of extreme political instability it is not the current policy of the current government that matters, but the subjective expectations regarding the future government and its policies (Goodhart 1998, King 2004). Iraqis expected the Coalition Provisional Authority to convert Hussein's money into its new legal tender, so there was no reason to reject it. Similarly, Russians' acceptance of the dead czar's money during the chaotic hyperinflation of the early 1920s (Friedman 1992, pp. 11-12) can also be explained by a belief that whoever ends up in power would either convert that money into a new money or accept it in various payments. The Germans' premature abandonment of their money can reflect expectations that the Allies would treat them harshly and not conduct such a conversion.[23] The fact that ex-post the Germans' expectations turned out to be wrong is irrelevant for the validity of the theory.

Evidence from regime transitions, therefore, does not contradict the theory. Moreover, it is hard to explain any currency substitution that happens near a regime transition without acknowledging the government's role. If the government and its laws really have no role, then we should expect the old money, that of the previous regime, to circulate permanently, rather than the new one. There are three reasons for this surprising conclusion. First, the old money has a limited supply, as its issuing regime is dead. By contrast, the new money might suffer from inflation because its issuer is alive (Friedman 1992, p. 12). Second, the new money is very likely to be inflated since the new regime has not yet established effective systems for taxation and loans, and thus needs inflationary finance (Friedman 1992, p. 191).

[23] Alternatively, perhaps they expected to be treated the way they treated some occupied peoples. It is well known what monetary theory predicts regarding fiat money in finite horizon economies.

Third, the people are used to accepting the old money rather than the new one. These three factors should give the old money an enormous advantage over the new money.

Yet the old money does not circulate except, perhaps, during a transitional period. Eventually the new money takes over in spite of its disadvantages. The theoretical conclusion that the old money must dominate is easily reversed when some government intervention is taken into account. The legal tender status of the new money can make the crucial difference between the two moneys. The old money's value can be based only on extrinsic beliefs, old habits, and adaptive expectations; the new money is supported by the tax-foundation mechanism. The evidence from regime transitions therefore strongly supports the tax-foundation theory (Goodhart 1998), even if the political transition differs in timing from the monetary transition.

7.1.3 Ineffective Tax System

The mechanism may fail if the tax system is not functioning (Wray 1998, p. 36). If agents in the model believe that they will not be punished in the near future ($\tau = 0$), then the expected punishment is zero even if the regime itself is expected to persist ($G > 0$). This is particularly true for new regimes because it takes them time to construct effective tax systems that will detect and punish tax offenders severely enough. This may explain the failure of Japanese money right after Japan occupied Taiwan in 1895 (Li and Wright 1998), and the problems of new moneys in disintegrated federations where tax collection was not important beforehand: the Confederacy during the American Civil War (Lerner 1956) and formerly communist countries. Recalling the similarity to convertibility, it is similar to a situation where the treasury's gold holdings are lost or expected to be lost. With no immunities to sell to the taxpayers, the money becomes unbacked.

The government may denominate the tax in one unit of account and accept another money as well. If it accepts the other money according to market value rather than at a pre-determined exchange rate, there is nothing to prevent the collapse of the other money, as shown in Proposition 3. There is always an equilibrium in which both the market and the tax authority see that money as valueless. This actually happened to two famous hyperinflated moneys: the continental currency of the American Revolution (Calomiris 1988, p. 59) and the *mandat* currency of the French Revolution (Nussbaum 1950, p. 50).

There are other ways in which hyperinflation can ruin tax system. First, the real tax burden drops because the late collection effect dominates the tax bracket effect. Even *daily* indexation of taxes as in the 1946 Hungarian hyperinflation cannot help because of the inevitable lag (Varga 1949). Second, monetary punishments are usually not indexed and thus no longer deter. Third, it may be better to incur the punishment rather than accept such money and starve. Finally, government revenues might drop so much that no government agency functions properly. Activities like auditing, prosecuting, conducting a trial, and executing the punishment are all seriously disturbed, making the risk of actually being punished

negligible. During the German hyperinflation, for example, the government was bankrupt and its tax system was destroyed (Keynes 1924, p. 68). Debtors optimally defaulted temporarily even in the face of certain loss in court. This clogged the courts with lawsuits and made such defaults even more profitable (Wolf 1993). The purchasing power of the Hungarian government was severely eroded by the lag from collection of tax revenues to their spending by the Treasury on government activities (Varga 1949).

7.1.4 Non-exclusive Circulation

In many countries the legal tender currency circulates side by side with foreign currency. It is especially common in periods of high inflation, in which people prefer a foreign, stable currency whenever possible. As shown in Proposition 6, when agents can hold more than \$1, sometimes they might choose to conduct transactions in other ways. They may do so as long as they already hold the minimum they need for tax payments in the domestic currency (Wallace 1979). This is an important constraint on the government's power and the extent to which it can encourage the use of its favorite money in trade. However, it does not completely contradict the theory. At least occasionally agents do demand the domestic currency because they need it for tax payments. It is probably this demand that keeps the price level finite even after people realize that their government is bent on hyperinflation.

7.1.5 Summary

Failures of famous paper moneys are consistent with the theory, when it is carefully modeled and interpreted. It is not an all-powerful mechanism. It has important limitations: the existence of other legal tender currencies, pessimism about the government's viability, a dysfunctional tax system, and hyperinflation.

7.2 Is Tax Receivability Necessary?

The high positive correlation between the existence of regimes and the success of their currencies raises an intriguing possibility. Perhaps some government intervention is *necessary* to support the value of money. If a certain type of paper money is neither supported by convertibility nor forced on all transactions, does it have to be receivable for taxes in order to circulate? Standard monetary models say that this is not the case: pure expectations can sustain the circulation of such money. However, as Prescott and Rios-Rull (2005) show, such monetary equilibria collapse once any agent can issue his own money. If you offer me such money, I should say "thank you for the idea" and issue one myself without giving you a good for your money. What really prevents each one of us, in reality, from starting our own system of intrinsically useless, inconvertible money? In many countries it is legal and requires nothing but audacity. Indeed, it is very easy to find references in the literature to currencies that were supposedly intrinsically useless,

inconvertible, and not legal tender. It is convenient to deal separately with societies that have a written legal code ("modern societies") and those that do not ("non-modern societies").

7.2.1 Modern Societies

Bank deposits and checks are not legal tender and are not convertible into goods or services. However, they are convertible by law into some legal tender. As shown in Subsection 4.2 the tax authority may or may not encourage the use of such inside money in trade and tax payments. Similar to checks are modern private banknotes in Scotland and Northern Ireland, which have never been legal tender. They too are legally convertible into some legal tender (Bank of England notes). Private, non-bank local moneys such as the Ithaca HOURS and Toronto Dollars are convertible into the issuers' goods and services or the domestic legal tender.

Some paper moneys did circulate without any legal status, but only for a short time. A recent example is that of the *creditos* in Argentina. Hundreds of such private moneys were issued and circulated briefly before they all collapsed (Colacelli and Blackburn 2009). This fast collapse and the large number of issuers are more consistent with the aforementioned Prescott-Rios-Rull result than with standard monetary models' prediction that such money can circulate based on pure expectations. Such a short-run failure of the theory is no more troubling than the fact that people join pyramid schemes all the time. As with the *creditos*, it is optimal to start your own scheme instead of joining someone else's scheme, but unlike the *creditos* the collapse is logically inevitable. If people keep falling for pyramid schemes, it is not surprising that they fall for the less illogical private money. In the long run, however, such patterns of behavior are not sustainable at the macro level.

Among government-issued currencies there are many false examples. The paper moneys of most English American colonies, Bank of England notes during the Napoleonic Wars, and notes issued by towns in the U.S. in the nineteenth century, have been cited as not having a legal tender status. The truth is that these currencies were legal tender for taxes but not for contractual debts. That is, their success is clearly explained by the tax-foundation theory. Similarly, Confederate money of the American Civil War, mentioned in the opening quote of this chapter, was legal tender for taxes and for debts to banks, but not legal tender for other contractual debts. Federal Reserve notes had been legal tender only for taxes while they were convertible (before the Great Depression). Recently, Governor of the Bank of England Mervyn King (2004) mentioned that Hussein's older money continued circulating in Kurdish Iraq from 1993 to 2003, even though its legal tender status had been revoked by Hussein. However, King's claim that this money was no longer supported by *any* government is incorrect. The money was declared legal tender by the Kurdistan Regional Government that actually controlled the area.[24]

[24] Private communication with Mr. Nijyar Shemdin, the U.S. Representative of the Kurdistan Regional Government, 01/12/2004.

An apparent difficulty lies in the fact that the U.S. dollar is valued even in places not subject to U.S. taxation. This is not really a problem. Foreigners hold the U.S. dollar because its stable value – according to the tax-foundation theory – is assured by the effectiveness of the IRS in the U.S. The issuing regime must have effective tax collection *somewhere* for its money to have any value. A government that has no genuine jurisdiction anywhere cannot pull it off. An example is governments in exile like the Polish government during World War II. A detailed discussion and references for the legal status of modern money are available in Goldberg (2007).

7.2.2 Non-modern Societies

Some of the most important monetary economists have argued that paper money requires nothing more than pure "trust." They have argued that legal status is irrelevant, and "proved" it by referring to "primitive" or ancient societies. The implicit assumptions in referring to such societies are: (i) they have no legal systems; (ii) objects that modern Western people consider to be intrinsically useless have also been considered as such by people living in these societies. With these assumptions in mind, economists have found it easy to provide false examples of moneys that have circulated only because of "trust." They have done so by focusing on the peculiar object used as money, ignoring unwritten local laws, customs and religion, and ignoring the possibility of different esthetic tastes in different cultures. The most notable cases are the stone money of the Island of Yap and seashells (cited by Friedman, Keynes, Samuelson, Tobin, and others). These examples are false since these moneys had both legal and religious status, and were considered highly valuable for subjective esthetic reasons. I discuss these cases in Goldberg (2005) and many less famous cases in Goldberg (2003). I show that none of these cases is a conclusive evidence for the existence of economists' "fiat money" – an object without intrinsic value or legal status.

7.2.3 Summary

It seems therefore, that if a currency has no other anchor (intrinsic value, convertibility, forced usage in spot transactions), it can circulate in the long run only if it is receivable for taxes. Of course, a scientific theory has to be refutable. The challenge here for the theory's opponents is to find an example of a currency that circulated for a long time without having *any* such anchor and had a high value. The latter qualification is meant to exclude cases of token money which were received as small change by buyers whose other option was not to receive change at all. Even small change may require some legal status. In the antebellum period Congress authorized the Mint to issue copper coins but refused to grant them any other legal status. Not surprisingly, the public rejected this money (Carothers 1930).

8 Conclusion

In the real world there is more to government-issued money than intrinsic uselessness and inconvertibility: its acceptance in tax payments is guaranteed. Generally, no other objects can be forced on the tax authority. This chapter uses a monetary search model to prove in a dynamic economy that receivability for taxes can make such money the general and unique medium of exchange. Other contributions are comparison of this mechanism to convertibility, exploration of its limitations, and a survey of its relation to monetary history.

There are many ways for a government to promote the circulation of its paper money. The ideal way would assure money-holders that their money could be put to good use, be robust to inflation in the sense that the money will not be completely abandoned, and also be easy and cheap to implement. The tax-foundation mechanism could very well prove ideal. Its assurance that the money can be put to good use was shown to be somewhat equivalent to convertibility. Its robustness to inflation was also shown: circulation can be maintained even while prices increase. As for implementation, this method is also the cheapest. There is no need to obtain and store gold. There is no need to monitor market transactions. There is no need to conduct searches for illegal currencies. Given that the government collects taxes in any case, it can easily promote any money simply by insisting on accepting only that money.

In fact, this theory explains that bare minimum that people continue to hold during hyperinflation. It is that bare minimum that makes inflationary finance successful at all. As noted by one of the most successful inflation fighters of recent history (Michael Bruno in Israel), it is the money needed for tax payments that prevents an immediate abandonment of money as soon as its issuer's intentions become apparent (Bruno 1993, p. 8). If the tax-foundation mechanism thus affects money demand then it affects the rate of inflation itself. More sophisticated models will need to quantify that relation.

As mentioned in the Introduction, the tax-foundation theory is part of Knapp's (1905) state theory of money. The German economist celebrated the fact that the government determined what was money and he disregarded issues such as the quantity of money and inflation. His theory was very popular in Germany in the early 20th century and was viewed by German policy-makers as a license to print money. The disastrous results are well-known. In contrast, the normative goal of this chapter is to promote an understanding of this menace and its limitations so that we can be better protected from it.

A final remark is about linguistics. In law and in any dictionary "fiat money" is intrinsically useless, inconvertible money *which is legal tender*. That would make sense because in English "fiat" means "order by a government." Most monetary theorists omit from this definition of "fiat money" the legal tender aspect or any other legal status. They talk about a "fiat component" or "fiatness" of money as referring to the aspects of money which are anchored in nothing but self-fulfilling expectations of its acceptance (as opposed to its intrinsic value), rather than its legal status which they usually ignore. One even

reads about "private fiat money." In mainstream journals there is no way to get around this linguistic travesty (e.g., the title in Goldberg, forthcoming). Here I avoided this term as much as possible to prevent confusion between the legal "fiat money" (discussed here) and the economists' "fiat money."

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