Micro-Foundations of the Keynesian Multiplier

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Abstract

The Keynesian multiplier, which is the foundation of Keynesian economics, is shown to be entirely consistent with strict micro-foundations. The key to the result is to use micro-foundations different from those of the neoclassical model. The multiplier, as developed by Kahn (1931), speaks of heterogeneous agents facing borrowing constraints who produce and consume different goods. These ingredients are exactly those found in the monetary search model of Kiyotaki and Wright (1989). I first show how a real, temporary sectoral shock generates a long, economy-wide business cycle. I then show how and when policies can help in the recovery.

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1. Introduction

The current global financial crisis and the unprecedented policy responses to it have revived interest in the ideas of John Maynard Keynes. Many economists, especially in academia, still oppose Keynesian economics because, so it is argued, it has no micro-foundations. Blaming the stagflation of the 1970s on this deficiency, these economists have entrenched themselves in highly mathematical models that do have strict micro-foundations: a Walrasian auctioneer, Arrow-Debreu markets, a utility-maximizing representative agent, a profit-maximizing representative firm, and a single good.

It should not be surprising that these models generally fail to produce Keynesian outcomes. The basic problem lies in the fact that the Keynesian multiplier, the foundation of Keynesian economics, exists only in circumstances which are very different from those micro-foundations of the neoclassical model. The basic story of the multiplier, as told by Richard F. Kahn in 1931 and as told to millions of undergraduate students ever since, describes a cash-strapped agent who receives money from the government. He uses some of this new money to buy from another cash-strapped agent, who in turn uses that extra money to buy from yet another agent, *ad infinitum*. Necessary ingredients in the story are: borrowing constraints, decentralized spot trade, many types of goods, agents who differ from each other in their production capabilities and consumption preferences, and a government that can selectively help some agents rather than all of them at once. All these ingredients are found in the search model of Kiyotaki and Wright (1989) and the literature that followed. These happen to be the conditions which make money essential.[1]

[1] Wallace (2001) summarizes the main contributions of that literature.

This paper uses a variant of the monetary search model to provide for the first time strict, mathematical micro-foundations to the Keynesian multiplier as originally formulated by Kahn. The main difference is that the search model explicitly takes into account the time lags between earning money and spending it. Kahn was aware of these lags but put them aside for analytical convenience, while Keynes thought they mattered only if it took producers time to adjust to an unexpected shock.[2] Kahn and Keynes thus derived a large *static* jump in output. The fact that the theory is static is still a major source of criticism.[3] In contrast, I derive a *dynamic* process where changes in purchasing power explicitly propagate across sectors and through time.

Insufficient demand is also key in Clower (1965) and Leijonhufvud (1968), but instead of disequilibria in Walrasian markets I use a non-Walrasian model. The paper is also related to the literature on sectoral linkages through intermediate goods (Long and Plosser 1983), in which a sectoral shock deprives other sectors of a production input.[4] Here, sectors are connected by liquidity from the demand side instead of the supply side. Kiyotaki and Moore (1997a) show sectoral linkages through the credit market, where a debtor's default causes a liquidity contagion because each agent owes money to its supplier. They derive an immediate contagion similar to the Keynesian multiplier. I show a liquidity contagion in an economy with no credit at all and I describe a complete business cycle. The business cycle in Kiyotaki and Moore (1997b) also has contagion but not due to illiquidity: it comes from changes in asset prices of collateral.

The paper proceeds as follows. Section 2 presents the basic environment and the steady state. Section 3 shows that a one-period negative shock to one sector propagates across sectors and

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[2] Kahn (1931), p. 183, fn. 2; Keynes (1936), Chapter 10, Section IV.

[3] For a very recent example see Levine (2009).

[4] Also see Scheinkman and Woodford (1994), Murphy, Shleifer, and Vishni (1989), Lilien (1982).

through time by consecutively affecting the purchasing power of different agents. The result is an economy-wide, long and auto-stabilizing business cycle. I show the robustness of this result to assumptions about divisibility (Section 4), shock properties (Section 5) and consumption preferences (Section 6), while generating more business cycle regularities. Only after this negative shock does it make sense to analyze policies. I show how monetary policies (Section 7) and fiscal policies (Section 8) might help in the recovery. Section 9 concludes.

2. The Basic Model

As mentioned above, the basic ingredients of a monetary search model are the ones needed to derive the Keynesian multiplier. I adopt the following standard features of such a model. Time is discrete and denoted t, with $t \in \{T, \ldots, 0, \ldots, \infty\}$. There is a continuum of infinitely-lived households, with a unit mass of each type $i = 1, \ldots, n$. There are n types of perishable goods, also denoted $i = 1, \ldots, n$. A type i household gets utility $u(q) > 0$ from consuming q units of good i+1 (mod n), and incurs a cost $c(q) > 0$ from producing q units of good i. The discount factor is β. The only durable object is intrinsically useless money. A type i household holds $m_t^i < 3$ money units (\$$m_t^i$) at the beginning of period t. The money distribution is M_t. Households are anonymous and trading histories are private information.

The monetary search model has well-known problems of analytical tractability. I therefore adopt the following approach. I will construct an economy in which the steady state is one of full employment, literally. The distribution of money in that steady state will be degenerate in an *endogenous* manner. That is, I will not force it to be degenerate at the end of every period as in Shi (1997) and Lagos and Wright (2005). Next I will show how a negative sectoral shock distorts the distribution of money until it realigns endogenously after a few periods. It is only in the context of such a recession that policies are interesting here. I show later how the government can help in the recovery back to the steady state.

The endogenous degenerate distribution of money in steady state is derived here by combining directed search with two-agent households, as suggested by Wallace (2002). The directed search model I modify is Goldberg (2007). Each type *i* household lives in a house in City *i*, which is where good *i* is produced. The collection of all such households is called Sector *i*. Each household consists of a buyer and a seller. The seller stays home, while the buyer travels with the household's money to City *i*+1. I assume away matching frictions within a city: all the buyers arrive first at the city's center and leave for the local houses sequentially in a random order. Each one picks a house randomly. No house has room for more than one buyer. Given the equal sizes of buyer and seller populations, every buyer finds one seller and every seller finds one buyer.[5]

As usual in models of this type the informational assumptions rule out credit and gift-giving, while the structure of production and consumption preferences prevents barter. The goods' perishability prevents commodity money, and thus the only alternative to autarky is use of the intrinsically useless money. The price and quantity in a match are determined by symmetric Nash bargaining as in Trejos and Wright (1995). When the buyer has $\$m_t^i$ and the seller's household has $\$m_t^{i+1}$, the former pays $\$d_t^i(m_t^i, m_t^{i+1})$ and the latter produces $q_t^{i+1}(m_t^i, m_t^{i+1})$ of good *i*+1, subject to participation constraints. The buyer then returns home, consumes the goods

[5] This is essentially an assumption that the short side of the market is fully matched (e.g., Matsui and Shimizu 2005). It has been justified elsewhere with a parable on taxicabs waiting outside an airport terminal for sequentially exiting passengers. Other real-life examples are the egg-sperm match in conception and the game musical chairs.

with its spouse and disposes of excess money such that $m_{t+1}^{i} < 3$. Household members cannot communicate with each other while in different locations. Let E_t be the expectations operator and let $V_t^i(m_t^i, M_t)$ be the value function of a type *i* household at the beginning of period *t*. Then

$$V_t^i(m_t^i, M_t) = u(q_t^{i+1}(m_t^i, m_t^{i+1})) - c(q_t^i(m_t^{i-1}, m_t^i)) + E_t \beta V_{t+1}^i(m_t^i - d_t^i(m_t^i, m_t^{i+1}) + d_t^{i-1}(m_t^{i-1}, m_t^i), M_{t+1}). \tag{1}$$

Note that the seller and buyer of a household simultaneously match with households of different types. The Nash bargaining rule finds the pair $(d_t^i(m_t^i, m_t^{i+1}), q_t^{i+1}(m_t^i, m_t^{i+1}))$ which maximizes

$$\left[u(q_t^{i+1}(m_t^i, m_t^{i+1})) + E_t \beta V_{t+1}^i(m_t^i - d_t^i(m_t^i, m_t^{i+1}) + d_t^{i-1}(m_t^{i-1}, m_t^i), M_{t+1})\right] \times \left[-c(q_t^{i+1}(m_t^i, m_t^{i+1})) + E_t \beta V_{t+1}^{i+1}(m_t^{i+1} + d_t^i(m_t^i, m_t^{i+1}) - d_t^{i+1}(m_t^{i+1}, m_t^{i+2}), M_{t+1})\right] \tag{2}$$

A buyer cannot pay more than it brings to the match (it cannot use its spouse's current income):

$$d_t^i(m_t^i, m_t^{i+1}) \le m_t^i. \tag{3}$$

The participation constraints for buying and selling, respectively, are

$$u(q_t^{i+1}(m_t^i, m_t^{i+1})) + E_t \beta V_{t+1}^i(m_t^i - d_t^i(m_t^i, m_t^{i+1}) + d_t^{i-1}(m_t^{i-1}, m_t^i), M_{t+1}) > E_t \beta V_{t+1}^i(m_t^i + d_t^{i-1}(m_t^{i-1}, m_t^i), M_{t+1}), \tag{4}$$

(5) $$-c(q_t^{i+1}(m_t^i, m_t^{i+1})) + E_t \beta V_{t+1}^{i+1}(m_t^{i+1} + d_t^i(m_t^i, m_t^{i+1}) - d_t^{i+1}(m_t^{i+1}, m_t^{i+2}), M_{t+1}) >$$
$$E_t \beta V_{t+1}^{i+1}(m_t^{i+1} - d_t^{i+1}(m_t^{i+1}, m_t^{i+2}), M_{t+1}).$$

Definition 1: A symmetric stationary monetary equilibrium is a collection of production quantities and payments in matches, and a distribution of money, such that: (i) The solution of the bargaining problem (2) subject to constraints (3)-(5), and taking the money distribution as given, is $d_t^i(m_t^i, m_t^{i+1}) = d > 0$ and $q_t^{i+1}(m_t^i, m_t^{i+1}) = q > 0$ for all i and t. (ii) The distribution is consistent with initial endowments and the evolution of money holdings implied by trades. (iii) The distribution is degenerate and constant: $m_t^i = m$ for all i and t.

To illustrate how the model works in the simplest possible way it is convenient to assume at first that both goods and money are indivisible, and that a household cannot produce or consume more than one goods unit each period. This is relaxed later. Let $u \equiv u(1)$ and $c \equiv c(1)$. The following assumption is necessary.

Assumption 1: $\beta^n u > c$ and $\beta u > (2+\beta)c$.

I first establish a degenerate distribution in steady state.

Proposition 1: If Assumption 1 holds and each household is initially endowed with \$1, then a symmetric stationary monetary equilibrium exists, in which each household both spends \$1 and earns \$1 each period.

All proofs are in the appendix. Each household ends every period holding \$1 so the degenerate distribution of money replicates itself. There is full (self-) employment. Output per household and the price level are 1, so with *n* types of unit mass each, aggregate output is *n*.[6]

[6] Obviously there is also an autarkic steady state in which nobody accepts money.

3. A Business Cycle

To the basic environment I now add n types of indivisible, perishable raw materials, denoted x^i, $i = 1, \ldots, n$. Production of any quantity of good i in period t now requires that an endowment of one unit of raw material x_t^i be available then to the producer. At any period, either all type i households get the endowment or none of them does (think of farmers and rain). Let X_t denote the distribution of raw materials, which is known to all households at the beginning of t. The value function is now $V_t^i(m_t^i, x_t^i, M_t, X_t)$ and the bargaining solution is ($d_t^i(m_t^i, x_t^i, m_t^{i+1}, x_t^{i+1})$, $q_t^{i+1}(m_t^i, x_t^i, m_t^{i+1}, x_t^{i+1})$). Assume that $x_t^i = 1$ for all i and all $t \neq \tau$. Only in period τ there is an unexpected, exogenous, negative, once-and-for-all, total shock to the endowment of one sector, lasting one period. That is, $x_\tau^i = 0$ for a particular i.[7]

The dynamic analysis is restricted to equilibria that satisfy long-run neutrality of the shock: eventually the economy goes back to the same steady state. From now on I focus on an example with $n = 3$, $i = 1$, and $\tau = 0$, unless noted otherwise. That is, only type 1 (out of three) is hit in period 0. The business cycle that follows this shock is described in Table 1 and Proposition 2.

Proposition 2: If Assumption 1 holds, then a one-period shock to Sector 1 results in a drop of aggregate output from 3 to 2 in periods 0, 1, and 2. It returns to the steady state level of 3 in period 3.

[7] Haberler (1963) and Mankiw (1989) emphasize large observable shocks as causing recessions. Shutting down the entire sector simplifies the analysis and makes it observationally equivalent to a total demand shock. It also emphasizes that the model is about shocks which may be too large to be handled in reality with standard financial contracts (those are ruled out here by the informational assumptions).

In period 0, type 1 buyers cannot smooth consumption due to the indivisibility of their only money unit (this is relaxed later). Although buying in period 0 will leave them with no money at all, it is better than saving it all for period 1 due to discounting. Since type 3 households cannot consume in period 0, if they produce that period they end up with \$2. They do because they know they will need the spare money later. Type 1 households cannot borrow from the rich type 3 households due to the anonymity assumption. For the same reason they could not have insurance contracts in advance.

In period 1, the production of good 1 can be resumed. Type 1 sellers do resume sales to type 3 buyers, but these proceeds arrive too late to be used for their spouses' shopping in period 1. Since type 1 households began the period without money they cannot buy good 2. Type 2 households, then, do not produce; they still spend all their money by buying from type 3 due to discounting and money's indivisibility, so now type 2 households are the ones who end up without money. Type 3 households buy and sell for \$1, so they still have \$2 each.

In period 2, type 2 households have no money so they cannot buy from type 3 households. The latter are the ones who accumulated a spare \$1 earlier. Now they spend it and end the period holding \$1, just like all other households. Anticipating this demand shock led them to accumulate a spare \$1 in period 0. At the end of period 2 the distribution of money is exactly as in the steady state. For the first time since the shock hit, everyone has both purchasing power and raw materials, so in period 3 the economy is back in steady state.

This is the basic mechanism that the paper explores. The trigger for the recession can be a supply shock or a demand shock that directly affects only one sector for only one period. But the recession does not last just one period. The shock impoverishes those making a living in the hit sector, and this brings down all other sectors, one by one, like domino blocks. While in the static

Keynesian multiplier all the domino blocks fall simultaneously – thus creating a large multiplier – here most sectors fall later. As Haberler (1963, pp. 231-232) notes, this lag must exist in a monetary economy, and it can be thought of as the time between paychecks.[8]

The return to full employment is "natural" (Haberler 1963, p. 265): once the original shock is gone, the sectors recover in the same order. The sector that was hit first accumulates new purchasing power that helps in the recovery of the next sector's purchasing power, and so on. This can be thought of as the Keynesian multiplier working in the positive direction. For the general case with n types, it can be conjectured that output is low by $1/n$ of its steady state value for n periods. Note that a type of a good, or a sector, can be interpreted as a geographical unit like a county or a country. Many real shocks, whether wars or disasters, are indeed local.

4. A Divisible Example

In the above example the indivisibility of both goods and money meant that prices could not decline in response to the slack demand and buyers could not smooth spending. Here I show that this exogenous rigidity may not drive the main results.

4.1. Divisible Goods

Assume first that goods are perfectly divisible and that the functions $u(q)$ and $c(q)$ have the usual properties as in Trejos and Wright (1995). Money is still indivisible for now. To trace the evolution of allocations during the transition I use a specific example. Let $u(q) = \sqrt[4]{q}$, $c(q)$ = .2492q, and $\beta = .998$.[9] Then (2) is maximized in steady state at $(d_t^i, q_t^{i+1}) = (1,1)$, as in Table 1.

[8] This is not the only possible pattern of recession. The non-monetary equilibrium may appear just for period 0. If the economy indeed completely shuts down for the duration of the shock, there is no propagation. The economy can return to the monetary steady state in period 1. This recession is worse than the one in Table 1 due to discounting.

[9] This high discount factor corresponds to the average time between paychecks in the U.S. (two weeks).

Given the assumed evolution of money holdings, I go back from the steady state which resumes in period 3 to period 0 to calculate output in each match by a recursive computation of the continuation values that appear in the bargaining problem.[10] As Table 2 shows, this exact calculation of prices gives an average gain of only .3% in accuracy of the quantities traded for $1, compared with the indivisible example (Table 1). The post-recovery stationary value function is much larger than any variation in periodic consumption or production during the transition. Thus, the Nash argument is maximized at almost the exact same level of output.

4.2. Divisible Money and Goods: The Equilibrium Remains

To verify that the equilibrium of Table 2 is robust to divisibility of money, suppose that somehow a type 2 buyer has $\$\varepsilon$ (where $0 < \varepsilon < 1$) instead of $0 at the beginning of period 2. It is easy to show as an extension of Proposition 1 that in the steady state in which all trades involve payments of $1, a subunit of money ($\$\varepsilon$) is valueless. Therefore, right before returning to steady state, in period 2, type 3 cannot be induced to produce for this $\$\varepsilon$. This means that a solution to the type 2 buyer's bargaining problem in period 1 does not involve carrying over such spare but useless money to period 2. That is, type 2 will not smooth its money spending even though its money can now be split. This in turn means, by the same logic, that type 2 will not accept a payment of $\$\varepsilon$ from type 1 in period 1. Therefore, type 1 will not carry over spare money from period 0. Given that no one else smoothes consumption, a deviating household cannot do it either. The equilibrium of Table 2 still exists.

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[10] This procedure, which takes advantage of the return to the steady state in finite time, assures that the equivalence between Nash bargaining and strategic bargaining remains valid. As shown by Coles and Wright (1998), in the general dynamic case Nash bargaining amounts to assuming that agents are myopic.

4.3. Divisible Money and Goods: A Smoothing Equilibrium does not Exist

A different question is whether there is another equilibrium, in which *everyone* smoothes consumption perfectly. I look for an equilibrium in which prices drop during the transition such that even though some payments are less than \$1 yet they all buy the stationary goods quantities. In such a case the shock does not propagate at all. Such an equilibrium requires that nobody ever runs out of money. Table 3 shows an example in which nobody runs out of money during the transition and nobody ends up with a useless $\$\varepsilon$ in steady state (note the different column headings). The argument below, however, is more general.

I restrict attention to equilibria in which – as in Table 1 – the stationary money distribution is restored at the end of period 2. Then in a period-2 match between a type 2 buyer and a type 3 seller the Nash argument is $\left[u(q_2^3)+E_2\beta V_3^2(1,x_3^2,M_3,X_3))\right]\left[-c(q_2^3)+E_2\beta V_3^3(1,x_3^3,M_3,X_3)\right]$. Recall that the subscript denotes time and the superscript denotes type. This is essentially the same argument as on the steady state path, and thus $q_2^3=1$. Indeed the price falls just right due to the lower nominal spending. Going back in time, it can be verified that indeed this stationary quantity is determined by Nash bargaining in all the other matches. But there is a problem.

Proposition 3: The smoothing equilibrium does not exist.

The problem is that such a plan violates type 3's participation constraint in period 0. Given that *any* quantity of money buys one unit of a good during the transition, and given that type 3 expects to earn money every period in the future, it cannot be induced to produce in period 0. There is no point in accumulating spare money early if just enough money will keep coming in forever (recall the discussion after Proposition 2). In the example of Table 3, type 3 could rest in period 0, and spend $.5 in both period 1 and period 2. Thus, this equilibrium does not exist.[11]

5. Other Shock Realizations

Going back to the indivisible case, I now analyze a shock that hits most sectors or lasts more than one period. When two sectors are hit at once the recession is shorter (see Table 4). The shock to Sector 2 prevents type 1 from buying so its money holdings are not depleted. Instead, type 2 immediately runs out of money, and in the following period, after failing to buy from the rich type 3, the recovery is almost complete. This case can represent a single shock that affects most sectors directly (e.g., an oil shock). The point is that even in such a case, the effects of the shock persist after it is gone and they hit other sectors. Although such a shock affects many agents directly, it does not affect everyone in the same way and at the same time. For example, a columnist is little affected by an oil shock at first, but may be fired later because of low newspaper purchases by those directly affected by the shock.

Consider now a long shock instead (and let households hold up to $3). Table 5 shows that the longer shock not only delays the recovery, as could be expected, but also causes an amplified, hump-shaped response of output, as in the data (Cogley and Nason 1995). In no period is there a shock to two sectors at once and the shock itself is not hump-shaped. Yet the output gap looks after a while as if two sectors are hit at once (i.e., aggregate output is 1 instead of 3).

6. Robustness to Preferences

Here I change, one at a time, features in the Kiyotaki-Wright consumption preferences.

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[11] If we update this conjectured path, type 2 will also not produce in period 0: given that type 3 refuses to sell, a production by type 2 will make *it* the one accumulating redundant money. Since type 1 cannot produce, the economy shuts down in period 0. But this outcome is not new: it is the non-monetary equilibrium of footnote 6.

6.1. Substitution in Consumption

If households could shift consumption from the shocked sector to another sector, would this offsetting prevent an aggregate business cycle? Assume that type 3 prefers good 1 but will try to consume good 2 if good 1 is unavailable. To facilitate such an alternative scenario, also assume that there can be positive shocks, so that a positively shocked sector can produce two units of a good and receive two buyers per seller. Specifically, in period 0 Sector 1 is hit by a negative shock, but Sector 2 is blessed with a positive shock.

Table 6 describes the result. Type 3 wants good 2 instead of good 1. Type 1 still wants good 2. All type 3 buyers and type 1 buyers visit City 2. With enough of good 2 to sell to both types, aggregate output is unchanged for the duration of the shock. But a recession follows *anyway*. The problem is the same as before! The distribution of money has been disturbed. Type 1 households cannot consume in period 1, just like before. The fact that the spare money is now held by type 2 rather than type 3 cannot prevent the recession. It does, however, shorten the recession, given this particular Kiyotaki-Wright linkage between sectors.

6.2. Uncorrelated Specializations

The Kiyotaki-Wright patterns of specialization are correlated: all the producers of good i are also the only consumers of good i+1. This is unrealistic: just because both you and I are economists does not mean that we both like chicken and non-economists do not. Assume then that half of type i households consume good i+1 and half consume good i+2.[12]

Figure 1 shows that the propagation mechanism is robust to this change. The recovery starts earlier but lasts longer than in Table 1. The drop in output spreads immediately across the entire

[12] Money is still useful because a type i buyer who likes good i+1 cannot identify type i+1 sellers who like good i.

economy, and yet the recession is long. This pattern of contagion is more realistic than the illustrative chain reaction assumed above, and should be adopted in empirical applications.

6.3. Consuming Two Types of Goods

Real people consume more types of goods than they produce. Assume that every period type i households want a unit of good i+1 and a unit of good i+2, can produce two units of good i, are endowed with $2 each, and can hold up to $3 each. In steady state each buyer spends $1 in each relevant city in a random order.

A shock to Sector i now results in consumption smoothing. Half of type i buyers buy only in City i+1 and the other half buy only in City i+2, and they all save half of their money for the next period. The result is an immediate drop in output of *all* three sectors: in Sector i due to the exogenous shock, and in Sectors i+1 and i+2 due to the demand shock from Sector i. This causes both an immediate amplification of the shock and sectoral comovement.

Figure 2 shows an example with n = 5 (the larger number of types is needed to maintain essentiality of money with the new consumption preferences). Note the immediate amplification, compared with the basic example in which each type consumes only one good. The reduced specialization in consumption speeds up the propagation across sectors, and thus reduces persistence. However, even with each household consuming a staggering 40% of all the goods types in the economy, persistence is still substantial. Figure 3 shows comovement of sectoral outputs. While in the basic example all cross-sector correlation coefficients are negative during the transition (equal to -1/(1+n)), here they range from -.3 to .95. Their average of .37 is close to the data (Long and Plosser 1987).

In conclusion, realistic changes in consumption preferences do not eliminate the propagation. Some of them even improve the model's ability to replicate business cycle features. Similarly, it does not matter if only some producers of a good are hit by a shock. While the remaining, functioning producers will become richer at the expense of consumers, the main result remains: whoever is hit reduces consumption of other goods, thus propagating the shock.

7. Monetary Policies

In the basic indivisible example of n = 3 and i = 1, the socially optimal solution to the shock's propagation is for type 3 households to give their money to type 1 households in period 0, even though they do not get goods in return. Without credit markets, they have no incentive to do so. Without private insurance or savings, only the government can help the economy. *Monetary policy* is defined here as any policy that adds new money and does not explicitly tax anyone. Such a policy can be helpful in that it gives money to the "weakest link" – the households who become impoverished by the shock. In doing this, it helps maintaining stable demand for the other goods. The exact way in which the money is added turns out to be very important. I assume that inflation has no costs, and for now I also assume that prices are flexible and the policy is unexpected.

7.1. A Proportional Transfer

Suppose that, right after the shock, at the end of period 0, the government unexpectedly increases all households' money holdings proportionately by 100%. In the long run, prices must be $2 to maintain the neutrality of money.

Lemma 1: There is an equilibrium in which all prices increase to 2 in period 1 and remain at that level.

Table 7 shows that nothing changes in real terms. Money is neutral even in the short-run, and the recession is just as long and as bad as before.

7.2. A Lump-sum Transfer

Suppose that, right after the shock, the government gives $1 to each household. Even with indivisibility of money and goods, this can still result in an equilibrium with an immediate price adjustment as above, under the following assumption: if all households of some type hold odd quantities of money but the price level is even, they all pool money within their type and redistribute it so as to maximize their type's consumption. For example, in period 1, when the price is 2, type 1 households pool their $1 and redistribute it in a lottery, such that half get nothing and half get $2 each.[13] On average, then, this type can purchase .5 units.[14] As shown in Table 8, this policy is helpful because it immediately restores some of type 1's purchasing power (half of it), thus reducing the magnitude of the propagation of the shock.

A $2 lump-sum transfer is even better, as shown in Table 9: with a new money supply of 9 units, the new price level is 3, and type 1 can buy 2/3 units in period 1 rather than 1/2. As seen in the previous tables, the economy is back at full employment if and only if the previous period ends with a degenerate money distribution. Given that the sectoral shock distorts the distribution, a lump-sum transfer can realign it immediately only if it is infinitely large. With arbitrarily small inflation costs this is clearly not optimal. However, this exercise hints that recovery may be better achieved if the policy is directed towards the hit sector. This possibility is explored next.

7.3. A Directed Transfer

Suppose that the government gives new money to those hit directly by the shock, such that the money supply increases by 100%. The results are in Table 10.[15] Such a directed monetary

[13] Allowing contracts only within each sector maintains the essentiality of money.

[14] Alternatively, with an assumption of divisible goods, all such households will buy .5 unit each.

[15] The aggregate results are only incidentally identical to those of a 100% lump-sum transfer (Table 8).

transfer may actually be a better approximation to real-world open market operations than the policies discussed above. This is because, when one sector is hit and its workers and owners lose wages and dividends, they are more likely than others to immediately sell their bond holdings for cash. The injection of new money, traded for bonds immediately after the shock, is then more likely to happen with agents in the hit sector. Although there is no official or designed policy of giving new money to the hit sector only, this is effectively what could happen, as demonstrated in a very different model by Alvarez, Atkeson, and Kehoe (2002).

To find the optimal size of the transfer, the policy-maker needs to take into account that resurrecting type 1's purchasing power with too much money decreases the purchasing power of others (through the increase in prices). In particular, type 1's purchasing power should not be made more than it can consume (1 unit), as described in Lemma 2 and Table 11.

Lemma 2: A better directed transfer would be 1.5, if there were no indivisibility issues.[16]

The closer the policy brings the distribution of money holdings back to a degenerate one the quicker is the return to steady state. Since the steady state has full employment, the quicker is the return the better. Therefore, the optimal monetary policy, shown in Table 12, would give $2 to type 1 and $1 to type 2, thus immediately realigning the money distribution and restoring the steady state. This is not because of some egalitarian considerations per se, but because the degenerate distribution happens to be the distribution of money in steady state. It is not clear how to implement such a policy in reality.

[16] Indivisibility is not an issue if steady state money holdings are already even, say because of a previous monetary expansion. If everyone already has $2, Lemma 2 means that the transfer and the new price level are both 1.5 times this, or $3.

By the way, a degenerate distribution in steady state is reasonable in some sense. Type *i* households represent everyone who makes a living in sector *i*, from the most junior employees to the CEOs and shareholders. Taken as a group, it is not clear whether any sector in the economy is significantly richer than another. In fact, free entry should rule out such a possibility.

As I show next, things become more complicated when one introduces sticky prices and policy expectations.

7.4. Policy Problems

Suppose that prices are exogenously sticky for one period. They have to be $1 in period 1 and can adjust only in period 2. As before, indivisibility is overcome with pooling, a lottery, and redistribution. If another redistribution is needed at some point, I assume that the former losers then become the winners.

Table 13 shows that under price stickiness, a proportional transfer is not neutral. Surprisingly, it is not harmless either, as we might expect from a proportional transfer. It prolongs the recovery and is worse than no intervention (Table 1) both in aggregate terms and for most types. Sticky prices are usually thought of as good for a monetary expansion because then the extra money increases purchasing power. The problem here is that a proportional transfer does not give money to those who need it the most, i.e., type 1 households. The other types already have enough money regardless of the price level. The new money, translated into increased purchasing power of the rich households because of stickiness, only messes up the auto-stabilizing money distribution.

Table 14 shows a lump-sum transfer with sticky prices. It is better than in the case of flexible prices (Table 8) and so better than non-intervention (Table 1). Sticky prices are therefore good here. It is easy to show that the same holds for the other monetary experiments in Tables 9-11.

Once the hit agents get \$1 somehow, then as long as the price level is 1 they can buy as they did before the shock. The economy gets a temporary break from the effect of the shock so long as the stickiness lasts. Once it is over, we get the full cycle as before. Due to discounting, and because agents here are not averse to the increased volatility, this is good. Stickiness does not matter for the optimal monetary policy of Table 12.

Further complications can arise if the policy is expected. Suppose that prices are sticky and a policy of a proportional monetary transfer is expected. This policy could harm the economy in a different way. Since it rewards households for high money holdings, type 1 households may choose not to buy during period 0. That way they keep their \$1 and get another dollar at the end of the period. With \$2 they can buy every period until the return to steady state. Thus they give up u in period 0 but gain $\beta u + .5\beta^2 u$ in periods 1 and 2 (compare with Table 13). If $\beta + .5\beta^2 > 1$ then it is optimal not to buy in period 0. This means that type 2 will not sell and thus it faces the exact same choice as type 1. If both types choose not to buy then the economy is effectively shut down in period 0 (see Table 15). In period 1 the economy is back at full employment with sticky prices and in period 2 there is still full employment but prices adjust. The loss of three units of output is similar to Table 1, only that most of the loss in Table 1 is discounted while here it is not. Therefore this policy is harmful.

It is easy to show that if the policy is expected but prices are flexible no one gives up consumption in period 0. Then the analogous condition is $\beta > 1$, which never holds. It is therefore the *combination* of policy expectations and price stickiness that causes this problem.

8. Fiscal Policy

Fiscal policy is defined here as a policy that taxes and then redistributes existing money. The redistribution can be a gift, or a wage for the production of some public works, whose utility is not modeled. Such a policy is another way of providing badly needed money to the “weakest link,” only that this time it comes from other taxpayers rather than the central bank. I ignore the possibility that this money is already at the Treasury or borrowed by it.

Suppose that at the end of every period the government randomly takes \$1 from a fraction $Z < 1$ of those households who have more than they need for current consumption, and gives all of that money to households in shocked sectors. In steady state there are neither rich households nor shocked households, so this tax law is moot. But after a shock to Sector 1 all type 3 households are eligible for such taxation. I assume here that the production cost is small enough such that type 3 households produce in period 0 even if they expect such probabilistic taxation. The recovery for the case $Z = .5$ is described in Table 16, where it is assumed that in period 2, when only half of type 3 sellers can sell, those who were taxed get the privilege of selling their goods.

This policy is successful. Applied to reality, it is like fiscal aid that troubled sectors sometime receive. Notable examples include “too big to fail” policies regarding banks, insurance companies and the car insdustry in 2008-2009 and the airline industry after September 2001. Since sectors in the model can be interpreted as geographical areas, the model somewhat justifies federal assistance to disaster areas by FEMA and presidential declarations of *disaster areas*, which make such areas eligible for fiscal benefits. Of course, in reality, moral hazard makes such policies problematic.

This policy is also observationally equivalent here to unemployment insurance because only those who do not work in period 0 get the aid in real time. It is also observationally equivalent here to food stamps, because only those who end period 0 with no money at all, and are about to go hungry in period 1, receive aid. It is easy to conceive of a general version of this model, where these fiscal programs will *not* be observationally equivalent to ad-hoc fiscal aid.

Ad-hoc fiscal aid requires legislation, and this could take time. The conventional wisdom is that such a policy lag can be harmful (a tax cut might stimulate the economy after it had already recovered, thereby causing an excessive expansion and a crisis later on). Here I show a different problem caused by a policy lag when a specific sector gets aid. As before, the shock is only in period 0. Legislation is written in period 1, and the aid (financed as before by progressive taxation) is given to Sector 1 at the end of period 1. As seen in Table 17, the policy lag delays the return to full employment compared with non-intervention.

The reason is that, when help arrives, Sector 1 had already recovered, and now the trouble is with Sector 2. Without government intervention (Table 1), the crisis is about to be over, because in the following period – period 2 – Sector 2 will transfer the shock to Sector 3, which is cushioned against the shock because it had previously saved money. With a policy lag, however, the propagated shock that is about to hit (the taxed) Sector 3 is not as diminished as in Table 16, but still bears its full magnitude. The problem of policy lag happens because a particular sector is marked for help, and when the help arrives it is actually needed by another sector. The problem does not exist in programs that look at those who are *currently* unemployed or without money – namely, unemployment insurance and welfare payments.

9. Conclusion

It is commonly argued, especially since 2008, that the neoclassical growth model is not useful for real-world policy analysis. Some even say that looking for micro-foundations has proven counter-productive. Actually, the only problem has been that the search for micro-foundations *did not go far enough*. Instead of replacing just the ad-hoc IS equation, as the neoclassical model does, the monetary search model also replaces the ad-hoc LM equation. Once the monetary nature of the economy is thus taken "seriously" (as that term is used by Neil Wallace), Keynes's basic intuition is easily restored.[17]

As Table 1 shows, the *only* problem in the economy at the end of period 0 is that the money is misallocated. The real shock is gone so everyone can produce again. The recession could be immediately over if only the spare money held by type 3 households was transferred to the poor type 1 households. The assumptions of anonymity and private trading histories preclude credit and insurance markets here, so private markets cannot help. While these assumptions are extreme, some of the most famous recessions involved real shocks that happened to hit the economy right after a financial crisis that undermined the financial system.[18] Extreme yet popular assumptions in the other direction, namely complete and perfect markets, make money entirely redundant. The existence of money in the real world proves that such assumptions are not a good description of the real world (Wallace 2001). If one assumed, in contrast, a Walrasian or a two-sector barter economy, again money would not be needed. In that case the economy would collapse completely while the real shock hits because the shocked households have no income at all. The recession would be over immediately once that shock is gone.

[17] Mitchell (1927, pp. 75-82) claimed that modern business cycles began when autarky gave way to money incomes generated from specialization in the production of a few goods, with these money incomes then used to buy other goods. Not every basic model of money generates the same pattern of propagation. In a standard two-period overlapping generations model a total negetive shock to the only good irreversibly eliminates the flow of money to the next generation and all those that follow. The monetary equilibrium ceases to exist forever.

[18] Examples include the Dust Bowl (after the Great Depression), the September 11 attacks (after the burst of the dotcom bubble), and the 2008 oil shock (after the burst of the housing bubble).

Where the markets fail, fiscal or monetary policy can, in principle, help by reallocating the money. Real-life complications such as price stickiness, fiscal policy lags, and policy expectations, can make policies harmful. Price stickiness should be better understood, as it can either help or harm, depending on the policy. Monetary economists should examine which of their modeled policies most resemble open market operations.

The simple, tractable framework presented here not only reconciles strict micro-foundations with the original Keynesian economics, but is also shown to be flexible enough to accommodate a large variety of types of shocks, real-life complications, and policies. Future work will need to bring the model closer to reality on various dimensions.

Table 1: The Simplest Business Cycle

Period	Sector 1			Sector 2			Sector 3			*Y*
	Sell	Buy	End $	Sell	Buy	End $	Sell	Buy	End $	
-1	1	1	1	1	1	1	1	1	1	3
0	**0**	1	0	1	1	1	1	0	2	2
1	1	0	1	0	1	0	1	1	2	2
2	1	1	1	1	0	1	0	1	1	2
3	1	1	1	1	1	1	1	1	1	3

Notation: "Sell" and "Buy" denote quantities of goods sold or bought by a household during that period. "End $" is the stock of money held by a household at the end of that period. *Y* is aggregate output. The bold "**0**" is the exogenous shock.

Table 2: Indivisible Money, Divisible Goods

Period	Sector 1			Sector 2			Sector 3			*Y*
	Sell	Buy	End $	Sell	Buy	End $	Sell	Buy	End $	
-1	1	1	1	1	1	1	1	1	1	3
0	**0**	1.004	0	1.004	1.008	1	1.008	0	2	2.012
1	.9961	0	1	0	1.012	0	1.012	.9961	2	2.008
2	1	1	1	1	0	1	0	1	1	2
3	1	1	1	1	1	1	1	1	1	3

Table 3: The Evolution of the Money Distribution with Smoothing

Period	Sector 1			Sector 2			Sector 3		
	Earn	Spend	End	Earn	Spend	End	Earn	Spend	End
-1	1	1	1	1	1	1	1	1	1
0	**0**	.5	.5	.5	1	.5	1	0	2
1	1	.5	1	.5	.5	.5	.5	1	1.5
2	1	1	1	1	.5	1	.5	1	1
3	1	1	1	1	1	1	1	1	1

Notation: All the numbers refer to units of money.

Table 4: An Almost-Aggregate Shock

Period	Sector 1			Sector 2			Sector 3			*Y*
	Sell	Buy	End $	Sell	Buy	End $	Sell	Buy	End $	
-1	1	1	1	1	1	1	1	1	1	3
0	**0**	0	1	**0**	1	0	1	0	2	1
1	1	1	1	1	0	1	0	1	1	2
2	1	1	1	1	1	1	1	1	1	3

Table 5: A Long Shock

Period	Sector 1			Sector 2			Sector 3			*Y*
	Sell	Buy	End $	Sell	Buy	End $	Sell	Buy	End $	
-1	1	1	1	1	1	1	1	1	1	3
0	**0**	1	0	1	1	1	1	0	2	2
1	**0**	0	0	0	1	0	1	0	3	1
2	1	0	1	0	0	0	0	1	2	1
3	1	1	1	1	0	1	0	1	1	2
4	1	1	1	1	1	1	1	1	1	3

Table 6: Substitution in Consumption

Period	Sector 1			Sector 2			Sector 3			*Y*
	Sell	Buy	End $	Sell	Buy	End $	Sell	Buy	End $	
-1	1	1	1	1	1	1	1	1	1	3
0	**0**	1	0	**2**	1	2	1	1	1	3
1	1	0	1	0	1	1	1	1	1	2
2	1	1	1	1	1	1	1	1	1	3

Table 7: Monetary Policy: A Proportional Transfer

Period	Sector 1			Sector 2			Sector 3			*P*	*Y*
	Sell	Buy	End $	Sell	Buy	End $	Sell	Buy	End $		
-1	1	1	1	1	1	1	1	1	1	1	3
0	**0**	1	0; 0	1	1	1; 2	1	0	2; 4	1	2
1	1	0	2	0	1	0	1	1	4	2	2
2	1	1	2	1	0	2	0	1	2	2	2
3	1	1	2	1	1	2	1	1	2	2	3

Notation: In "End $" column, "*j*; *k*" means that at the end of trade the household had $*j* and after policy intervention it has $*k*. *P* is the aggregate price level.

Table 8: Monetary Policy: A Lump-sum Transfer

Period	Sector 1			Sector 2			Sector 3			*P*	*Y*
	Sell	Buy	End $	Sell	Buy	End $	Sell	Buy	End $		
-1	1	1	1	1	1	1	1	1	1	1	3
0	**0**	1	0; 1	1	1	1; 2	1	0	2; 3	1	2
1	1	.5	2	.5	1	1	1	1	3	2	2.5
2	1	1	2	1	.5	2	.5	1	2	2	2.5
3	1	1	2	1	1	2	1	1	2	2	3

Table 9: Monetary Policy: A Larger Lump-sum Transfer

Period	Sector 1			Sector 2			Sector 3			*P*	*Y*
	Sell	Buy	End $	Sell	Buy	End $	Sell	Buy	End $		
-1	1	1	1	1	1	1	1	1	1	1	3
0	**0**	1	0; 2	1	1	1; 3	1	0	2; 4	1	2
1	1	.67	3	.67	1	2	1	1	4	3	2.67
2	1	1	3	1	.67	3	.67	1	3	3	2.67
3	1	1	2	1	1	2	1	1	2	3	3

Table 10: Monetary Policy: A Directed Transfer

Period	Sector 1			Sector 2			Sector 3			*P*	*Y*
	Sell	Buy	End $	Sell	Buy	End $	Sell	Buy	End $		
-1	1	1	1	1	1	1	1	1	1	1	3
0	**0**	1	0; 3	1	1	1	1	0	2	1	2
1	1	1	3	1	.5	2	.5	1	1	2	2.5
2	.5	1	2	1	1	2	1	.5	2	2	2.5
3	1	1	2	1	1	2	1	1	2	2	3

Table 11: Monetary Policy: A Better Directed Transfer

Period	Sector 1			Sector 2			Sector 3			*P*	*Y*
	Sell	Buy	End $	Sell	Buy	End $	Sell	Buy	End $		
-1	1	1	1	1	1	1	1	1	1	1	3
0	**0**	1	0; 1.5	1	1	1	1	0	2	1	2
1	1	1	1.5	1	.67	1.5	.67	1	1.5	1.5	2.67
2	1	1	1.5	1	1	1.5	1	1	1.5	1.5	3

Table 12: Optimal Monetary Policy

Period	Sector 1			Sector 2			Sector 3			*P*	*Y*
	Sell	Buy	End $	Sell	Buy	End $	Sell	Buy	End $		
-1	1	1	1	1	1	1	1	1	1	1	3
0	**0**	1	0; 2	1	1	1; 2	1	0	2	1	2
1	1	1	2	1	1	2	1	1	2	2	3

Table 13: Monetary Policy: A Proportional Transfer with Stickiness

Period	Sector 1			Sector 2			Sector 3			*P*	*Y*
	Sell	Buy	End $	Sell	Buy	End $	Sell	Buy	End $		
-1	1	1	1	1	1	1	1	1	1	1	3
0	**0**	1	0; 0	1	1	1; 2	1	0	2; 4	1	2
1	1	0	1	0	1	1	1	1	4	1	2
2	1	.5	2	.5	.5	1	.5	1	3	2	2
3	1	1	2	1	.5	2	.5	1	2	2	2.5
4	1	1	2	1	1	2	1	1	2	2	3

Table 14: Monetary Policy: A Lump-sum Transfer with Stickiness

Period	Sector 1			Sector 2			Sector 3			*P*	*Y*
	Sell	Buy	End $	Sell	Buy	End $	Sell	Buy	End $		
-1	1	1	1	1	1	1	1	1	1	1	3
0	**0**	1	0; 1	1	1	1; 2	1	0	2; 3	1	2
1	1	1	1	1	1	2	1	1	3	1	3
2	1	.5	2	.5	1	1	1	1	3	2	2.5
3	1	1	2	1	.5	2	.5	1	2	2	2.5
4	1	1	2	1	1	2	1	1	2	2	3

Table 15: Monetary Policy: An Expected Proportional Transfer with Stickiness

Period	Sector 1			Sector 2			Sector 3			*P*	*Y*
	Sell	Buy	End $	Sell	Buy	End $	Sell	Buy	End $		
-1	1	1	1	1	1	1	1	1	1	1	3
0	**0**	0	1; 2	0	0	1; 2	0	0	1; 2	1	0
1	1	1	2	1	1	2	1	1	2	1	3
2	1	1	2	1	1	2	1	1	2	2	3

Table 16: Fiscal Policy

Period	Sector 1			Sector 2			Sector 3			*P*	*Y*
	Sell	Buy	End $	Sell	Buy	End $	Sell	Buy	End $		
-1	1	1	1	1	1	1	1	1	1	1	3
0	**0**	1	0; .5	1	1	1	1	0	2; 1.5	1	2
1	1	.5	1	.5	1	.5	1	1	1.5	1	2.5
2	1	1	1	1	.5	1	.5	1	1	1	2.5
3	1	1	1	1	1	1	1	1	1	1	3

Table 17: Fiscal Policy with a Lag

Period	Sector 1			Sector 2			Sector 3			*P*	*Y*
	Sell	Buy	End $	Sell	Buy	End $	Sell	Buy	End $		
-1	1	1	1	1	1	1	1	1	1	1	3
0	**0**	1	0	1	1	1	1	0	2	1	2
1	1	0	1; 1.5	0	1	0	1	1	2; 1.5	1	2
2	1	1	1.5	1	0	1	0	1	.5	1	2
3	.5	1	1	1	1	1	1	.5	1	1	2.5
4	1	1	1	1	1	1	1	1	1	1	3

Figure 1: Changes in Aggregate Output when Specializations are Uncorrelated

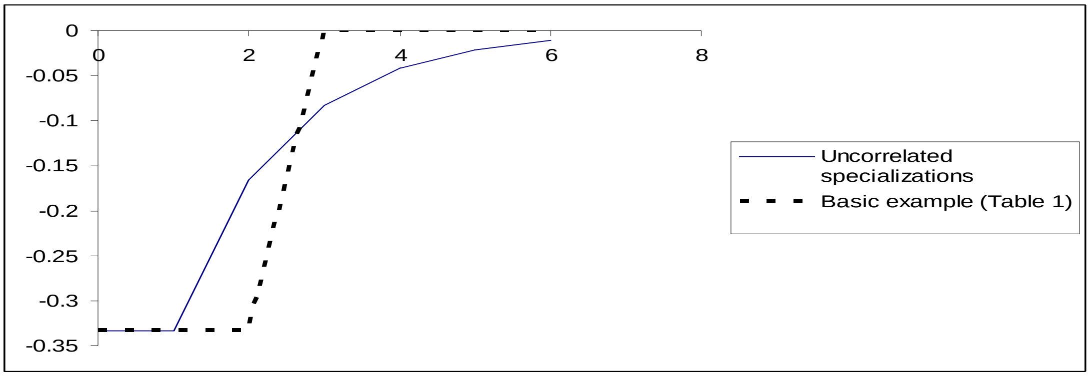


Figure 2: Changes in Aggregate Output when Consuming Two Different Goods

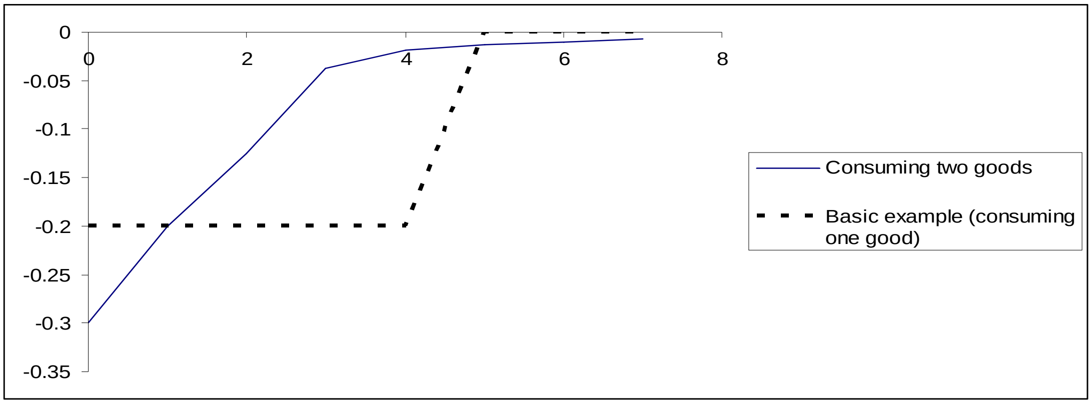
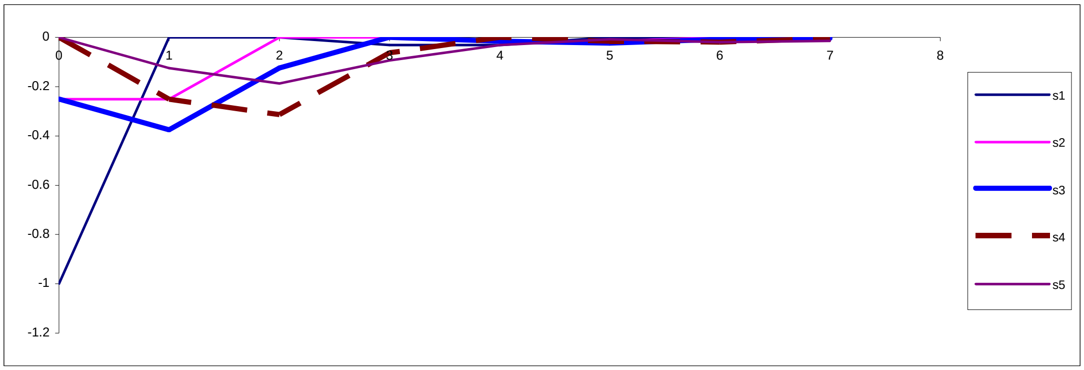


Figure 3: Changes in Sectoral Outputs when Consuming Two Different Goods



Notation: s1 is sales (and production) of good 1.

Appendix

To ease on the notation I omit here aggregate state variables in the value functions.

Proof of Proposition 1: Starting from the initial period *T*, the proof rolls forward, period by period. It uses conjectures about future periods to verify conjectures about earlier periods. The general conjecture is that *every period* any household who can buy does spend \$1, and any household with less than \$2 produces and gains \$1 from selling. Thus it is conjectured that any household ends the period with \$1 regardless of how much money it had at the beginning of the period. That is, $V_t^i(m_t^i) = Au - Bc + E_t\beta V_{t+1}^i(1)$, where A = 0 for m_t^i = 0 and A = 1 otherwise, and B = 0 for m_t^i = 2 and B = 1 otherwise. Initially all households have \$1, so $m_T^j = 1$ for all j =1, …, n in (2). Also, the conjecture implies $d_T^{i-1}(1,1) = d_T^{i+1}(1,1) = 1$, so (2) becomes $\left[u(q_T^{i+1}(1,1)) + E_T\beta V_{T+1}^i(2 - d_T^i(1,1))\right]\left[-c(q_T^{i+1}(1,1)) + E_t\beta V_{T+1}^{i+1}(d_T^i(1,1))\right]$.

The indivisibility assumptions imply that only one interesting solution may satisfy both participation constraints. In this monetary solution $(d_T^i(1,1), q_T^{i+1}(1,1)) = (1,1)$ so the outcome is $\left[u + E_T\beta V_{T+1}^i(1)\right]\left[-c + E_T\beta V_{T+1}^{i+1}(1)\right]$, provided that $u + E_T\beta V_{T+1}^i(1) > E_T\beta V_{T+1}^i(2)$ and $-c + E_T\beta V_{T+1}^{i+1}(1) > E_T\beta V_{T+1}^{i+1}(0)$. Given the conjecture about period *T*+1, the former constraint becomes $u + E_T\beta[u - c + E_{T+1}\beta V_{T+2}^i(1)] > E_T\beta[u + E_{T+1}\beta V_{T+2}^i(1)]$ or $u > \beta c$. Similarly, the latter constraint becomes $\beta u > c$. The key is that the conjectures involve the same continuation value $E_{T+1}\beta V_{T+2}^i(1)$, which cancels out in the constraints. Thus, the monetary solution satisfies the constraints. If the constraints hold then $\left[u + E_T\beta V_{T+1}^i(1)\right]\left[-c + E_T\beta V_{T+1}^{i+1}(1)\right] >$ $\left[E_T\beta V_{T+1}^{i+1}(2)\right]\left[E_T\beta V_{T+1}^{i+1}(0)\right]$, so the monetary solution is better than the non-monetary solution and it maximizes the Nash argument. This outcome for every match means that each household indeed holds \$1 at the end of period *T*.

Just as I used here the period *T*+1 conjecture to verify the period *T* conjecture, so can the period *T*+2 conjecture be used to verify the period *T*+1 conjecture, and this can be repeated forever. However, off the equilibrium path households' money holdings at the beginning of any period but *T* might be \$0 or \$2. If a buyer has \$2, a payment of only \$1 (rather than \$2) maximizes the Nash argument iff

$\left[u + E_t \beta V_{t+1}^{i}(1)\right]\left[-c + E_t \beta V_{t+1}^{i+1}(1)\right] > \left[u + E_t \beta V_{t+1}^{i}(0)\right]\left[-c + E_t \beta V_{t+1}^{i+1}(2)\right]$. It holds if $\beta u > (2+\beta)c$. Other off-equilibrium matches do not require any more conditions. Thus the steady state is constructed, period by period. *QED*

Proof of Proposition 2: The proof is similar to that of Proposition 1 but much longer due to the non-stationary, non-symmetric episode. I show here only the most complicated, non-trivial, and critical case: Type 3 households accumulating spare money in period 0. The sale satisfies their constraint iff $-c + E_0 \beta V_1^3(2, x_1^3) > E_0 \beta V_1^3(1, x_1^3)$. I conjecture that they keep buying and selling in period 1: $V_1^3(m_1^3, 1) = u - c + E_1 \beta V_2^3(m_1^3, x_2^3)$, for $m_1^3 > 0$. Then the constraint becomes $-c + E_0 \beta^2 V_2^3(2, x_2^3) > E_0 \beta^2 V_2^3(1, x_2^3)$. Because here the value functions do not cancel out, I need an extra conjecture: In period 2 they cannot sell so $V_2^3(m_2^3, 1) = u + E_2 \beta V_3^3(m_2^3 - 1, x_3^3)$, for $m_2^3 > 0$. Then the constraint becomes $-c + E_0 \beta^3 V_3^3(1, x_3^3) > E_0 \beta^3 V_3^3(0, x_3^3)$. Period 3 conjectures are as in the steady state: $V_3^3(m_3^3, 1) = m_3^3 u - c + E_3 \beta V_4^3(1, x_4^3)$ for $m_3^3 < 2$. Then the constraint becomes $\beta^3 u > c$, so type 3's constraint holds in period 0. It is easier to show that type 2's constraint also holds in this match, and thus the monetary solution is chosen. Together with the results of the other period-0 matches, it implies that at the end of period 0 the money distribution is as in Table 1. This result is useful for proving the other conjectures. *QED*

Proof of Proposition 3: Type 3's participation constraint in period 0 is $-c + E_0 \beta V_1^3(1+d_0^2, x_1^3) > E_0 \beta V_1^3(1, x_1^3)$. This equilibrium requires buying and selling of one goods unit in period 1, regardless of money holdings. The money holdings at the end of period 1, however, may depend on money holdings at the beginning of period 1. Let m_2^{*3} denote money holdings on this equilibrium path. Then $V_1^3(1+d_0^2,1) = u - c + E_1 \beta V_2^3(m_2^{*3}, x_2^3)$ and $V_1^3(1,1) = u - c + E_1 \beta V_2^3(m_2^3, x_2^3)$. The period-0 constraint then becomes $-c + E_0 \beta^2 V_2^3(m_2^{*3}, x_2^3) > E_0 \beta^2 V_2^3(m_2^3, x_2^3)$. In period 2, buying and selling of one goods unit continues, but at the end of period 2 the household will hold \$1 either way: $V_2^3(m,1) = u - c + E_2 \beta V_3^3(1, x_3^3)$ for any $m > 0$. The constraint is then $-c + E_0 \beta^3 V_3^3(1, x_3^3) > E_0 \beta^3 V_3^3(1, x_3^3)$, and it obviously does not hold. *QED*

Proof of Lemma 1: Suppose that everyone expects prices to rise to 2 immediately. Given that sellers expect all prices to be even forever, none of them would agree to be paid \$1, since this would be as useless as being paid \$0. *QED*

Proof of Lemma 2: A transfer of \$*T* to type 1 increases the long run price level to *P* = *MV*/*Y* = (3+*T*)*1/3, where *M* is aggregate money, *V* is velocity, and *Y* is aggregate output. Given the immediate price adjustment, the transfer immediately increases type 1's purchasing power to *T*/*P* = 3*T*/(3+*T*). This exactly restores type 1's purchasing power to 1 if and only if *T* = 1.5. Then *P* = 1.5. *QED*

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