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Money with partially directed search ☆

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Abstract

Partially directed search replaces the total randomness of monetary search models. Agents choose whether to stay in their production location or visit other locations. Each visiting agent randomly chooses one shop among many in each location. As in random matching models, a commodity or fiat object can endogenously become money, but the details are richer and conform better with evidence: any commodity can be money; the ‘‘best’’ commodity is the most likely money; fiat money can totally crowd out commodity money in an asymmetric environment; going shopping is more likely than door-to-door sales. The model nests Walrasian equilibria.

JEL classification: E40; D83

Keywords: Directed search; Money; Markets; Shops

1. Introduction

The random matching model has become the workhorse of decentralized monetary economics. While many of the technical restrictions of Kiyotaki and Wright (1989, henceforth KW) have been relaxed in the subsequent literature, one major restriction inherently remains in all the random matching models. This is the random matching

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property itself. Agents cannot choose whom to meet or where to be. They behave as "particles colliding in space".[1] Random matching is very tractable but it is unrealistic. Modern real-life trade is almost as far from the total chaos of random matching as it is far from the perfect coordination of the Walrasian framework. This may be one major reason that the results of random matching models are still treated with some suspicion by many economists.

One may wonder what is the nature of monetary equilibria when there is *partially directed search*, as in real life, rather than total randomness. *Partially directed search* has a specific meaning here. It means that the search process has two stages—a directed stage and a random stage. In the directed stage agents deliberately and actively get access to potential sellers of the good they desire. Then, when they face many ex-ante identical sellers, comes the second stage: they randomly choose which particular seller to visit. Such a search process is a reasonable characterization of many trade activities. For example, someone who needs a taxi cab can look in the Yellow Pages under "Taxi" and then randomly choose one of those listed there. Going to the marketplace or the mall, or typing "shoes" in Yahoo! are other examples of the first step in this search process. The second, random stage keeps money essential.

The comparison between partially directed search and random matching is facilitated by using a model which is otherwise identical to KW. Agents have only one more piece of information, and choose only one more strategy, beyond that which they have and choose in KW. They know where all the fixed production locations are, and choose in which *specific* production location to be. In equilibrium, it is endogenously and simultaneously determined who offer goods in their production locations (i.e., open shops), and which objects become money.

It turns out that the main results of the standard random matching models are preserved: a commodity or fiat money can arise endogenously in equilibrium. However, the current model has three sets of useful results, in which it improves on random matching models. One set of results has to do with properties of the monetary equilibria. The relations between commodity money and fiat money, and between various potential commodity moneys, conform better with evidence from monetary history. Another set of results yields predictions about geographical trade patterns and relations between choices of shops and choices of money, which cannot even be discussed in a random matching model. Finally, there is much more order in equilibrium. The model therefore nests Walrasian equilibria, which realize if all agents happen to meet in the same place, many of its results easily generalize to any number of types of goods and agents, and most of its results do not depend on the particular pattern of specialization in production.

To facilitate the presentation of the numerous results the paper proceeds gradually. In Section 2 the basic economy is presented. Unlike KW, it has complete symmetry across goods. This model is analyzed in Sections 3 (autarky), 4 (Walrasian equilibria) and 5 (commodity money). Fiat money is added in Section 6. Section 7 gets closest to KW and reality by assuming that goods have different intrinsic properties. Section 8 concludes. All the proofs are in the working paper (Goldberg, 2006a).

[1] Burdett et al. (1995).

2. The model

To facilitate the comparison between a partially directed search model and a random matching model, for both commodity and fiat money, the model and notation are as similar as possible to Kiyotaki and Wright (1989). The description begins with those features of the model that are identical to KW, and is followed by the new features.

2.1. Standard features

This is a discrete time infinite horizon economy with indivisible goods 1, 2, and 3. There is a continuum of infinitely lived agents with unit mass. There are equal proportions of agents of types 1, 2, and 3. Agents specialize in both consumption and production: type i agents consume only good i and produce only another good i^*. Specifically, assume for now that $i^* = i + 1 \pmod 3$.[2] Agents derive utility U from consuming one unit of their desired good and disutility D from producing one unit of their production good, with $u \equiv U - D > 0$. Production must be preceded by consumption so with an initial endowment of one good per agent no one ever holds more than one unit of any good. Let V_{ij} be the value function of a type i agent holding good j. The discount factor is $\beta \in (0, 1)$.

2.2. The geographical environment

To introduce partially directed search in a tractable way, the following geographical environment is assumed. Every agent owns a structure that has several functions. First, it is his home. This is where he must be at the beginning and the end of every period, where the period may be thought of as a day. Second, it is his factory. This is where he produces his production good at the end of the day. Third, it is his storage place. He can store there his production good or any other good. Fourth, it *may* function as a shop. This is the only place in which he can offer visitors any object for trade. All the factories of type i agents are located in the same place, called City i.[3] This is the only place where their common production good i^* can be produced. All agents know where these cities are located and which goods are produced in each city. This piece of information is the critical difference between this model and KW.

Agents not only know more than in KW, but they also have the ability to act on that additional information. Every morning each agent chooses which city to be in for the day. He can stay home and open a shop (then he is called a *stayer*). Alternatively he can go to another particular city (then he is called a *searcher*). The travel cost for any searcher, from his factory to someone else's factory or back, is represented by the weight of the good he carries. Good i weighs $c_i \geqslant 0$.[4] In Sections 3–6 it is assumed that $c_i = c\ \forall i$. Stayers do not incur any such transport costs, nor do they have any storage costs. By evening all searchers must go back to their homes, regardless of whether they traded or

[2] This is the structure of KW's Model A. In Section 7, where the goods differ from each other, I also discuss KW's Model B. In Sections 3–6 it does not matter which structure is assumed.

[3] In the literature locations are referred to as "islands" when the spatial separation *restricts* information. In contrast, here the existence of fixed natural production locations *enhances* information. The word "city" should also remind that there are infinitely many factories in each location.

[4] With the interpretation that KW's agents move all the time like particles in space, their storage costs can be thought of as transport costs.

not. Successful searchers can consume on the spot, so they do not incur transport costs on the way back.

One parable is of a primitive economy in which all goods are natural resources, and therefore must be produced in fixed specific locations: fish are produced in the fishermen's village near the lake, vegetables are produced in the farmers' village in the field, and meat is produced in the hunters' village in the plain. It is reasonable to assume that such an economy eventually converges to a steady state where this information is common knowledge and costless.[5] Another parable is the Yellow Pages, where all producers of a good are merely listed together under the same category. In both cases, a searcher has free information that enables him to direct his search toward that group of producers.

2.3. Matching

All type i searchers who visit City j arrive there together. They face infinitely many factories around them. They do not know which factories are open as shops and what good is actually offered in each shop.[6] Since all factories are ex-ante identical every searcher chooses randomly which factory to visit. This random choice of a factory represents the second stage in the partially directed search process described above. In the model it serves to preclude bilateral credit, so trade must be quid pro quo as in KW.

In order to maintain the bilateral matching of KW as much as possible, it is assumed that the short side of the market is fully matched. That is, if there are fewer searchers than factories, each factory is visited by one searcher at most.[7] If there are fewer factories than searchers, each factory is visited by at least one searcher. Two searchers of the same type never reach the same factory.[8] This rule in an economy of three types implies that each factory, whether it is open as a shop or not, is visited by zero, one, or two searchers.

When a shop is visited by two searchers, the stayer can trade with one of the searchers or both. If he is indifferent between them he conducts a lottery. A searcher must return home after visiting one factory, regardless of the outcome of that match, and even if the factory he chooses is not open as a shop at all. He can try again the next day. In order to clearly distinguish between random matching and partially directed search, it is assumed that searchers can trade with each other only when a stayer is involved in the exchange. They cannot trade in his shop if he is not there, they cannot trade behind his back, and they cannot meet outside shops.

2.4. Strategies and equilibrium

The strategies are more complicated than in random matching models because an agent chooses both a trading strategy (whether or not to exchange goods) and a specific location

[5] As a modern illustration, energy companies know that a lot of oil is produced in the Middle East, although they did not pay anyone to get this information, and the oil producers currently do not advertise their locations.

[6] Giving them such information will make the model more realistic, but will also take it further away from random matching models—and this is not useful for the purposes of this paper.

[7] A parable of this matching rule, which has been used elsewhere, is that of taxi cabs that wait in line in front of an airport terminal. Each passenger that exits the terminal goes to the cab which is currently the first in line.

[8] Since the proportions of types are identical, there are always enough factories for this matching rule.

strategy within a partially directed search setting.[9] The lack of a social arrangement that creates a marketplace for all goods in a certain location, requires agents to guess where they can find the goods that they want, and how they can pay for these goods.[10] There are three possible location strategies. The value function of a type i agent who holds good j at the end of the period is

$$V_{ij} = \beta \max[\text{value of going to City 1, value of going to City 2,}\\ \text{value of going to City 3}].$$

For a type i agent, "going to City i" equals "staying in City i" as no transport costs are incurred. The equilibrium concept is otherwise the same as in KW.[11]

Definition 1. A Nash equilibrium is a set of pure and stationary trading strategies, pure and stationary location strategies, and a stationary distribution of goods, such that: (i) each agent chooses strategies to maximize expected utility given others' strategies and the distribution; (ii) given the strategies, the result is that distribution.

Following KW, it is assumed that agents who are indifferent between trading and not trading do not trade. In the same spirit, it is assumed that agents who are indifferent between staying home and searching stay home.[12]

In KW storage costs are unavoidable. This may result in negative value functions, and may induce an agent to forgo consumption in order to avoid the possibility of subsequently holding his very costly-to-store production good forever. Their Assumption A and Lemma 1 address these concerns. Lemma 1 below shows that these problems do not exist with the transport costs assumed here, because they can be avoided by staying home.

Lemma 1. (i) *Value functions are non-negative*; (ii) *agents always accept their consumption goods and consume them immediately*.

Finally, if an agent holds a good that turns out to function as money, then he is called a *buyer*; otherwise, he is called a *seller*.

3. Autarky

There is a possibility that location strategies imply no matching at all, and thus an autarkic equilibrium. Consider the case where no agent chooses to be a searcher. If this is an equilibrium agents will never meet each other and will always hold their production goods. However, construction of the equilibrium requires a complete specification of

[9] Burdett et al. (1995) let agents choose between standing still and wandering around. The latter (costly) option increases the probability of randomly meeting other agents. They analyze only fiat money.

[10] Going back to the parable of a primitive economy, a farmer who wants to consume fish can go to the fishermen or wait for them to come to him. He can also speculate that they might have used the fish to buy meat from the hunters, in which case he may visit the hunters. This is different from trading post models (e.g., Shapley and Shubik, 1977), where a *pre-determined* pair of objects is traded at any post. In these models it is not clear how posts are created, how their rules are enforced, and how agents learn about them.

[11] Corbae et al. (2003) analyze *cooperative* endogenous matching. They also focus on comparison with KW, and reach results which are different from both KW and the *non-cooperative* partially directed search that is presented here.

[12] These assumptions can be justified by an arbitrarily small cost of not being idle.

strategies. Suppose then that agents accept only their consumption goods, and that both this trading strategy and the location strategy above apply no matter what the agents hold.

If a type 1 agent believes that everyone else follows these strategies, his optimal strategy is defined by the following equations:

$$V_{12} = \beta \max[V_{12}, -c + \max(V_{12}, V_{13}) - c, -c + V_{12} - c], \tag{1}$$

$$V_{13} = \beta \max[V_{13}, -c + V_{13} - c, -c + u + V_{12}]. \tag{2}$$

In (1), an agent who holds good 2 has three options. If he stays home nothing happens because nobody visits him. If he goes to City 2 he incurs a transport cost on his way there. Since he is the only searcher around and type 2 agents accept his good 2, he can get good 3 for sure, if he wants to. Whether he decides to trade or not, he incurs another transport cost when he goes back home. The last option is to go to City 3, where type 3 agents live. On his way there he incurs a cost c. Since type 3 agents accept only good 3 he cannot trade and he takes good 2 back home. In (2), he cannot trade in City 2 because everyone there already has the good 3 he brings. If he goes to City 3, he is the only searcher there and the locals want his good 3. Therefore, he gets good 1 for sure, consumes it on the spot, goes back home, and produces another unit of good 2. The next step is to guess that the agent follows the same strategies as everyone else, and then try to verify the guess. It turns out that it is optimal to stay home iff the transport costs are too high.

Proposition 1. *Autarky with no searchers is an equilibrium iff $u \leqslant c$.*

The same result appears in Burdett et al. (1995), for the same reason. It is important to note that this equilibrium's trading strategies in themselves do not imply autarky, as demonstrated in the Walrasian equilibria of the next section.[13] The only problem is the location strategies.

The opposite location strategies do not lead to equilibrium.

Proposition 2. *Autarky with no stayers is not an equilibrium.*

If one believes that everyone else are searchers, it is optimal to stay home. This is the only chance of meeting anyone, and it saves on transport costs.

In fact, a stronger statement can be made.

Proposition 3. *Autarky with any searchers is not an equilibrium.*

In equilibrium agents' expectations regarding others' behavior are correct. If an agent expects that others' behavior imply autarky, there is no point in searching.

4. Walrasian equilibria

Suppose that all type 1 agents and all type 3 agents take their production goods to City 2 and all type 2 agents stay there. Suppose also that they all accept only their consumption goods. Then in each shop there are a type 2 stayer, a type 1 searcher, and a type 3 searcher. There is no need for money: each type i agent gives his production good (good $i+1$) to the type $i+1$ agent (mod 3), and they all consume. The matching frictions are completely

[13] If it is assumed that agents who are indifferent do trade, then there is a similar autarky equilibrium in which agents agree to accept all goods.

overcome since the agents happen to coordinate on the "marketplace". Of course, the same can happen in any other city.

Proposition 4. *The three Walrasian equilibria exist iff* $u > c$.

As in Proposition 1, the transport costs must be low enough to enable trade. None of these equilibria Pareto dominates the others because each type is better-off in the equilibrium in which his city is the marketplace.

5. Commodity money

Suppose that type 3 agents take their production good (good 1) to City 2 and trade it for good 3. Suppose further that type 2 agents then take that good 1 to City 1 and trade it for good 2. If this happens in equilibrium then good 1 functions as *commodity money* and type 2 agents function as *intermediaries*, as in KW's *fundamental equilibrium*. Moreover, this equilibrium can be defined as a *shopping equilibrium*: both type 2 agents and type 3 agents trade by taking the commodity money to shops that sell their consumption goods. In other words, all searchers are buyers and all stayers are sellers. The opposite case is when type 2 agents go to City 3, take good 1 from there, and are then visited in City 2 by type 1 agents. Good 1 is still commodity money and type 2 agents are still intermediaries, but now the agents who trade good 1 for another good do so while in their homes. Thus, this is defined as a *door-to-door equilibrium*. There is only one other possible trading pattern in which good 1 is the unique money, and this is when type 2 agents, the intermediaries, are the only ones who leave their homes: they go first to City 3 and, after resting overnight at home, they then go to City 1. This is defined as a *one-searcher equilibrium*. The opposite of this equilibrium is when only type 2 agents stay home, but this is the same Walrasian equilibrium analyzed above. See Fig. 1 for an illustration of all these trade patterns. The case of more than one commodity money is discussed later. It turns out that only one of these types of commodity money equilibria can exist.

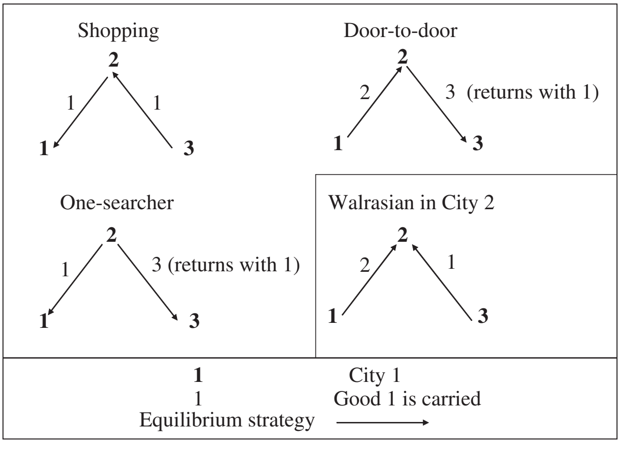


Fig. 1. Commodity money and Walrasian equilibria.

Proposition 5. *The shopping equilibria exist iff* $u > 3c$.

To understand this condition, consider again the case where good 1 is the candidate money. In the steady state of the shopping equilibrium, half of type 2 agents are in City 2, visited by all type 3 agents, and half of type 2 agents are in City 1, visiting all type 1 agents. Therefore, in City 2, where good 3 is produced, the demand for good 3 is twice the supply. A type 3 agent that visits City 2 incurs a cost c on his way there. With probability .5 he buys and consumes. With probability .5 he visits a factory whose owner is in City 1. Then he goes back home with the same good he brought. Thus, a visit to City 2 yields an expected payoff of $-c + .5u - .5c$, so it is optimal to visit City 2 iff $u > 3c$.

Proposition 6. *The door-to-door and one-searcher equilibria do not exist.*

In these equilibria the producers of the candidate money (type 3 agents) stay home. Thus, as opposed to the shopping equilibrium, they are accessible to type 1 agents. A type 1 agent is supposed to get good 1 from a type 2 agent in these equilibria, but if a type 2 agent deviates and offers him good 3, it is optimal for him to accept it. The reason is that he can take it to City 3, where he will be on the short side of the market and matched for sure (all type 3 agents are there, offering good 1, and the only other visitors are half of type 2 agents). If he waits for a type 2 agent to obtain good 1 for him then he is on the long side of the market (half of type 2 agents hold good 1, and all type 1 agents want to match with them). Given that type 1 agents accept good 3, type 2 agents do not need to get good 1 from type 3 agents in order to trade with type 1 agents. This means that good 1 is not money. Note that if a type 1 agent accepts good 3 from a type 2 agent (in order to take it to City 3), then good 3 becomes money instead of good 1. In the competition between goods on the role of money, there is a presumption against the one that is offered in shops by its producers.

For the same reason as in the Walrasian equilibria, no shopping equilibrium Pareto dominates another. Due to the matching frictions the shopping equilibria are dominated in every possible way by the Walrasian equilibria.

Proposition 7. (i) *If any shopping equilibrium exists, then all the Walrasian equilibria exist too*; (ii) *the shopping equilibrium with good i as money is Pareto dominated by the Walrasian equilibrium in which all trade is in City i*; (iii) *aggregate welfare is lower in any shopping equilibrium than in any Walrasian equilibrium.*

In KW most equilibria have two moneys but this cannot happen here.

Proposition 8. *There is no equilibrium with two commodity moneys.*

For example, for both goods 1 and 3 to be money, there must be meetings between type 1 agents and type 2 agents to make good 3 money, and meetings between type 2 agents and type 3 agents to make good 1 money, when all hold their production goods. With pure, stationary location strategies of type 2 agents all these meetings are in City 2, and this is the Walrasian, money-less equilibrium described above.[14]

[14]The phenomenon of multiple commodity moneys was observed in many societies. It can be easily generated in this model not only by making strategies mixed or non-stationary, but also by increasing the number of types of goods and agents. An example of two commodity moneys with four types is available by request. In general, with N types, it is probably the case that no equilibrium has $N-1$ moneys.

6. Fiat money

Here fiat money (good 0) is added to the economy. It is indivisible and has a transport cost $c_0 \geqslant 0$. A fraction $m \in (0, 1)$ of agents of each type are endowed with one unit of fiat money instead of a real good. The analysis is restricted to equilibria in which fiat money totally crowds out commodity money.

Definition 2. A fiat equilibrium is an equilibrium in which: (i) fiat money is used in all transactions; (ii) agents never accept real goods that they do not consume.

As in the commodity money equilibria, a *shopping* fiat equilibrium is one in which all buyers are searchers and all sellers are stayers. There are $1 - m$ sellers of each type who stay home with their production goods and m buyers of each type who take their fiat money and go shopping in the city that produces their consumption goods.

Proposition 9. (i) *A shopping fiat equilibrium exists for some parameter values*; (ii) *a necessary condition for existence is* $c_0 \leqslant c$.

Fiat money must be the lightest object for exclusive circulation. To see why, consider a type 1 stayer. He is supposed to trade only with a searcher who holds good 0, which he will later take to City 3. If $c_0 > c$ he prefers to do the same with good 3 instead. Knowing that, a type 2 agent will visit him with his production good (good 3) instead of going through the trouble of obtaining fiat money first.

The excessive traveling in this equilibrium has negative implications, regardless of how light the fiat money is.

Proposition 10. (i) *The shopping fiat equilibrium does not Pareto dominate any non-autarkic equilibrium*; (ii) *the shopping fiat equilibrium has lower aggregate welfare than any other non-autarkic equilibrium that coexists with it.*

The Pareto result stems from the fact that in any commodity money equilibrium and any Walrasian equilibrium there is at least one type of agents that never moves, while in the shopping fiat equilibrium all agents move. This result is robust to the particular assumptions on the matching process. However, the aggregate welfare result does depend critically on the assumed matching process. If agents could direct their search inside a city towards the open shops, and $m = .5$, there would have been no matching frictions in the fiat equilibrium and then it could have been better than all other equilibria. Even with the original assumption, fiat money can still be helpful if the parameter values are such that the only alternative is autarky.

In a door-to-door fiat equilibrium all agents take their production goods to their consumers, accepting only fiat money (and, off-equilibrium, their consumption goods); when they get fiat money they take it home, and there they accept only their consumption goods.

Proposition 11. *The door-to-door fiat equilibrium does not exist.*

It is optimal for everyone to deviate from such an equilibrium. For example, a type 2 agent who deviates by staying home with good 3, his production good, still gets good 2 from the type 1 searcher visiting him. That searcher agrees to trade good 2 for good 3, because the goods are equally heavy and good 3 will be accepted by the type 3 searcher who will visit him next period.

7. The asymmetric case

This section eliminates the symmetry across goods by assuming $c_1 < c_2 < c_3$. Good 1 now has the best intrinsic properties. The new results are better suited to a comparison with KW (and reality) than the preceding results. Some changes are obvious. Propositions 1 and 4 are replaced by the following.

Proposition 12. *Autarky with no searchers is an equilibrium iff* $u \leqslant c_1$.

Proposition 13. *A Walrasian equilibrium centered in City j exists iff* $u > \max(c_j, c_i)$, *where i is not type j's production good.*

In a Walrasian equilibrium both of the goods that are not produced in City j (good j is one of them) must be light enough to be transported to City j.

There are nine conceivable equilibria with commodity money: the three types of equilibria defined in Section 5, multiplied by the three possible commodity moneys in each equilibrium type. The difference in transport costs gives the model some degrees of freedom so more equilibria can exist than in Section 5. Propositions 5 and 6 are replaced by the following.

Proposition 14. (i) *All the shopping equilibria exist for some parameter values*; (ii) *the door-to-door equilibrium exists for some parameter values with either good* 1 *or good* 3 *as money, unless* $\beta \geqslant .5$ *or indifferent agents accept commodity money*; (iii) *the one-searcher equilibrium exists for some parameter values with good* 1 *as money, and is less likely to exist if indifferent agents accept commodity money*; (iv) *no other commodity money equilibrium exists.*

The fragility of the door-to-door equilibrium is significant as in any reasonably calibrated model $\beta > .5$.

Proposition 14(i) has a very important implication.

Theorem 1. *Any good can become money even when some goods have better intrinsic properties than others.*

In spite of this indeterminacy, there is a prediction regarding which equilibrium is more likely to exist than others.

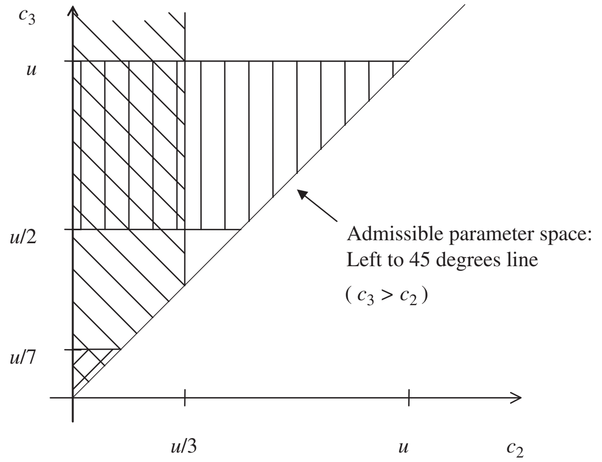
Proposition 15. (i) *The set of parameter values under which the shopping equilibrium with good* 1 *as money exists is a superset of the sets of parameter values under which all the other commodity money equilibria exist*; (ii) *only the shopping equilibrium has a strict monotonicity in the following sense*: *if it exists with good x as money, then it also exists with good y as money for all y that satisfy* $c_y < c_x$.

Fig. 2 demonstrates both parts of the proposition for $\beta = 1$ and $c_1 = 0$. Part (ii) follows from the fact that the shopping equilibrium is the only one in which only the money is transported (see Fig. 1).

The model thus gives a plausible explanation for three facts about the history of commodity money:

Fact 1: Trade was typically conducted by "going shopping".[15]

[15] Door-to-door sales were invented as an organized form of trade in the U.S. only in 1906 (http://www.salesdoor2door.com).



Necessary and sufficient equilibria conditions

Equilibrium \ Money	1	2	3
Shopping	All admissible space	$c_2 < u/3$ ╲╲╲	$c_3 < u/7$ ╱╱╱
One-searcher	$u/2 < c_3 < u$ │││	Do not exist [Proposition 14]	
Door-to-door	Do not exist [Proposition 14]		

Fig. 2. Shopping with the best commodity money is the most likely equilibrium.

Fact 2: Usually the commodity that was the most suitable to function as money due to its physical properties indeed functioned as money.[16]

Fact 3: Notwithstanding Fact 2, important deviations did occur. Almost any pre-industrial commodity functioned as commodity money at some place at some time.[17]

Welfare results are ambiguous.

Proposition 16. *Among the commodity money equilibria no type of equilibrium is unambiguously the best or the worst.*

The shopping equilibrium is efficient in the sense that all successful buyers carry nothing home (they consume on the spot). The one-searcher equilibrium is efficient in the sense that all searchers are successful: nobody incurs a transport cost in vain because of matching frictions. Combining these considerations with the ranking $c_1 < c_2 < c_3$ leads to ambiguous welfare ranking.[18]

Walrasian equilibria no longer dominate all commodity money equilibria.

Proposition 17. (i) *The set of parameter values under which the shopping equilibrium with good* 1 *as money exists can be a superset of the sets of parameter values under which all the Walrasian equilibria exist*; (ii) *this shopping equilibrium is not necessarily Pareto dominated*

[16]See the detailed survey by Einzig (1966).

[17]Again, see Einzig (1966).

[18]E.g., among equilibria with good 1 as money, the shopping equilibrium is better than the one-searcher equilibrium if $2c_1 < c_3$, even though it has more matching frictions (see the discussion after Proposition 5).

by any Walrasian equilibrium; (iii) *aggregate welfare can be higher in this shopping equilibrium than in one of the Walrasian equilibria.*

The shopping equilibrium has the unique advantage that only one good—potentially the lightest one—is ever carried in it, while any Walrasian equilibrium requires transporting two types of goods.[19] This can explain why money has existed even in very small societies that probably could have organized Walrasian exchange.[20] Interestingly, this preference for money is not necessarily a bad one. It turns out that the asymmetry between goods not only explains why some goods are more likely than others to be money, but also why monetary exchange can be more likely than, and better than, Walrasian exchange.[21]

The propositions above regarding autarky, Walrasian equilibria, and commodity money, imply the following.

Theorem 2. *For some parameter values the shopping equilibrium with good* 1 *as money is the unique equilibrium.*

This model allows any object to be traded at any location (unlike trading posts models). This is not only permissible; in fact, almost anything can happen in any city in equilibrium.

Proposition 18. (i) *For every pair* (n, h), *where* $n \in \{1, 2, 3\}$ *is a good and* $h \in \{1, 2, 3\}$ *is a city, there is a commodity money equilibrium in which good* n *is traded in City* h; (ii) *for almost every triple* (n, l, h), *where* $l \in \{1, 2, 3\}$ *is another good, there is an equilibrium in which good* n *is traded bilaterally for good* l *in City* h; (iii) *in every city there is an equilibrium with a trilateral exchange of all goods*; (iv) *for every city there is at least one equilibrium in which nothing is traded there.*

In part (ii), "almost" means eight out of nine possible exchanges. Parts (iii) and (iv) are corollaries of Propositions 13 and 12, respectively. Of course, not all these equilibria coexist for given parameter values.

Before proceeding to fiat money, it is time to recall KW's other production structure, called Model B (as opposed to Model A, which is the one analyzed so far). There type i produces good $i^* = i - 1 \pmod 3$. In KW the results differ substantially between Models A and B,[22] and the reasons for this difference have not been clarified. In contrast, all the propositions and theorems of this paper apply for Model B as well, with very minor changes.

Proposition 19. *For Model B*, (i) *in Proposition* 14(ii), *replace* "*either good* 1 *or good* 3" *with* "*good* 2;" (ii) *in Proposition* 14(iii), *replace* "*good* 1" *with* "*either good* 1 *or good* 2;" (iii) *Proposition* 15(ii) *no longer holds*; (iv) *the* "*almost*" *in Proposition* 18(ii) *means seven out of nine possible exchanges.*

[19] E.g., if good 2 is much heavier than good 1, then the shopping equilibrium with good 1 as money dominates the best Walrasian equilibrium (in which the two lightest goods—goods 1 and 2—are transported).

[20] Einzig (1966) shows that independent monetary systems have existed on a very large number of tiny Pacific islands.

[21] Obviously the assumption of consumption on the spot is critical for this proposition.

[22] In Model A there are two equilibria, with either good 1 as a unique money, or with goods 1 and 3 as money, and for some parameter values there is no equilibrium; in Model B there is always an equilibrium with goods 1 and 2 as money, and for some parameter values there is also an equilibrium with goods 2 and 3 as money. See KW's Theorems 1 and 2. Among other things it means that the best good (good 1) is the most likely money only in Model A.

Part (i) of the proposition hints at the correspondence between partially directed search and random matching. There is a striking similarity in the role of good 2 in the door-to-door equilibria and in KW. In both cases good 2 is the only good which is not money in Model A (Proposition 14(ii); KW's Theorem 1) but the most likely money in Model B (Proposition 19(i); KW's Theorem 2). The source of the similarity is seen in Fig. 1: the door-to-door equilibrium is the *only* non-Walrasian equilibrium in which the consumer of the commodity money is a searcher (and thus incurs transport costs). Moreover, he can meet agents of any other type because there are stayers of both types 2 and 3. This is as close as the current model can get to KW, where the costs are storage costs—always incurred by everyone—and agents can be matched with agents of any type. Another manifestation of this relation is the fact that all door-to-door equilibria and all KW's equilibria do not exist with symmetric transport costs.

As for fiat money, Proposition 9(ii) is modified in the obvious way.

Proposition 20. *A shopping fiat equilibrium exists only if* $c_0 \leqslant c_1$.

The fact that the shopping fiat equilibrium survives the generalization to asymmetric transport costs is worth emphasizing.

Theorem 3. *Fiat money can entirely eliminate commodity money in an asymmetric environment.*

The real world is asymmetric. Some goods, such as gold, are better than others as money. Such commodity moneys dominate monetary history but were entirely crowded out by fiat money. This model can generate this outcome, while random matching models cannot (Aiyagari and Wallace, 1992). Kultti (1994) and Burdett et al. (1995) do get equilibria where only fiat money circulates, but only by crowding out *direct* barter in the *symmetric* environment of Kiyotaki and Wright (1993).

The door-to-door fiat equilibrium still does not exist.[23] A result worth emphasizing follows from the propositions above.

Theorem 4. *Trade is much more likely to occur by "shopping" than by door-to-door sales, for both commodity money and fiat money.*

This dominance of "shopping" conforms well with historical evidence.[24]

The model is also easier to generalize than random matching models.

Proposition 21. *The results regarding autarky, Walrasian equilibria, and fiat equilibria hold for the general model with more than three types of goods and agents.*

[23] Now the only types who deviate for sure are those whose production goods are lighter than their consumption goods (i.e., type 3 in Model A, types 2 and 3 in Model B). The other types deviate for some parameter values, because the production good they may offer to a searcher still offers the marketability advantage discussed after Proposition 11.

[24] U.S. sales by interactive direct marketing (in person or by phone) are only 8% of GDP (Direct Marketing Association, 2002). Most of that is undoubtedly attributed to the existence of homebound homemakers and retirees, who are not modeled here. Also see footnote 15. A rare case of door-to-door sales being exceedingly popular is that of Russia in the early 1990s. The shortages in shops, typical of the Communist era, persisted for a while. Trade liberalization allowed people for the first time to import goods and sell them door-to-door to make up for these shortages. When the shortages were gone, so did the popularity of door-to-door sales. I thank Alexei Deviatov for this information.

This means that there is no need to resort to symmetry across goods or to their perishability to derive a shopping fiat equilibrium with many types.[25]

8. Conclusion

Total randomness in matching is not necessary for money to be essential. To preclude bilateral credit it is sufficient that a given pair of agents will meet again with probability zero. This is achieved in the current model by a random choice of a particular shop among the many shops that offer the desired good. Any more randomness than this, while mathematically convenient, can come at a cost in terms of the quality and quantity of results.[26] At the other extreme, a well-organized system of deterministic trading posts is also not necessary. This semi-random model may be thought of as a missing link between random matching models and trading post models.

As in random matching models, a commodity or fiat object can endogenously function as money, but the details are more consistent with monetary history. First, any good can be a unique commodity money. Second, the good with the best intrinsic properties is the most likely to become commodity money. Third, fiat money can totally crowd out commodity money, even when some goods are more fit to be commodity money than others. These results hold for both production structures of Kiyotaki and Wright (1989).

Other interesting results are obtained by the richer structure of this model: agents might coordinate on Walrasian, money-less equilibria; the best commodity money equilibrium can be more likely to exist than any Walrasian equilibrium; agents are more likely to carry commodity/fiat money to shops where their consumption goods are held (''go shopping''), than to wait at home with the money for door-to-door salespersons; fiat money never Pareto dominates commodity money and Walrasian equilibria.

The model is kept as close as possible to Kiyotaki and Wright (1989) to show in the clearest possible way the differences between random matching and partially directed search. Future work should incorporate extensions as those made to random matching models, such as endogenous prices, divisible money, mixed strategies and direct barter. It is also possible to combine partially directed search and random matching in the same model by assuming that a searcher has a positive probability of randomly meeting other searchers between shops.

A benefit of the approach taken here is that all equilibria have much more order and discipline than in random matching models. It allows generalizing most of the results to the case of more than three types of goods and agents, even when goods have different intrinsic properties. More important are the implications for using the model to analyze broader macroeconomic issues. For example, combining this model with an assumption that the decision-making unit is a two-agent household, as suggested by Wallace (2002), creates a degenerate distribution of money in steady state. Such a model can be used to analyze long-run monetary policy (Goldberg, 2006b) and the propagation of real sectoral shocks (Goldberg, 2006c). The choice of a particular location can also be extended to include a

[25]Regarding the non-existence of the door-to-door fiat equilibrium, now *only* those whose production goods are lighter than their consumption goods do deviate. For a searcher visiting them, the good he brought and the one he is offered are equally marketable, so only the transport costs matter.

[26]Wallace (2002) defends the random matching assumption as one possible source of idiosyncratic uncertainty about consumption and earning opportunities. This uncertainty produces heterogeneity of money holdings, which is important for some issues in monetary economics.

choice of deliberately going to the bank, the bonds market, the foreign exchange market, or the tax authority.

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